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ADTRANS
ADVANCED DATA ANALYSIS FOR TRANSPORTATION AND MOBILITY ACADEMIC TEACHERS

Parametric Significance Tests – homework

Joanna Wachnicka

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Parametric Significance Tests – Homework

Task 1

The technical standard provides for an average of 55 seconds for the performance of a certain technical operation by employees at a certain administrative position. Since employees complained that this standard was bad, timing measurements were made for n = 60 randomly selected individuals and an average of x = 72 seconds and s = 20 sec was obtained from this sample. Is it possible to reject the hypothesis at the significance level $\alpha = 0.01$ that the actual average time for performing this operation is in accordance with the standard?

Solution

Data:

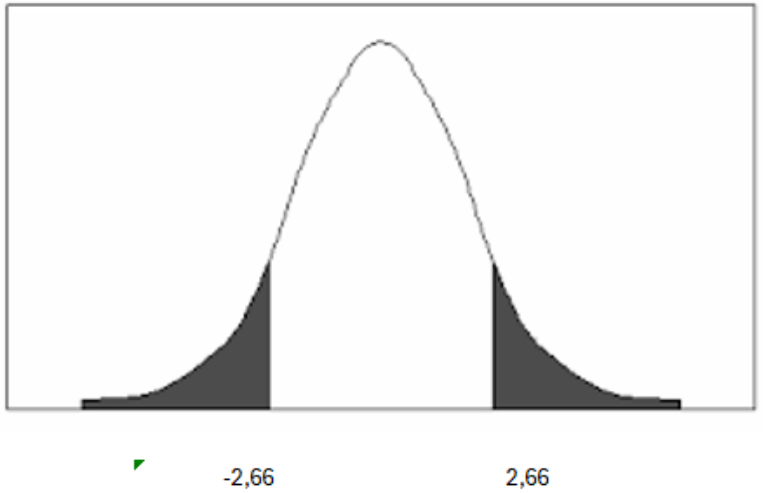
- $n = 60$
- $\alpha = 0.01$
- $\bar{x} = 72$
- $s_x = 20$

Hypotheses:

- $H_0: m_0 = 55$) null hypothesis)
- $H_1: m \neq 55$) alternative hypothesis)

Test Statistic Calculation:

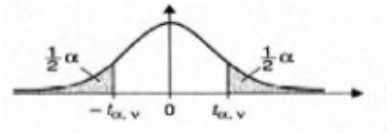
$$t_e = \frac{72 - 55}{20} \cdot \sqrt{60} = 6.528$$



A normal distribution curve with critical values marked at -2.66 and 2.66, showing the rejection regions at both tails, and the calculated test statistic t_e far outside the right tail.

Answer: There are grounds to reject hypothesis H_0 .

***T-Student* distribution**



$P(|t| \geq t_{\alpha,\nu}) = \alpha$

ν \ α	0,2	0,1	0,05	0,02	0,01	0,002	0,001
1	3,078	6,314	12,706	31,821	63,657	318,309	636,619
2	1,886	2,920	4,303	6,965	9,925	22,327	31,599
3	1,638	2,353	3,182	4,541	5,841	10,215	12,924
4	1,533	2,132	2,776	3,747	4,604	7,173	8,610
5	1,476	2,015	2,571	3,365	4,032	5,893	6,869
6	1,440	1,943	2,447	3,143	3,707	5,208	5,959
7	1,415	1,895	2,365	2,998	3,499	4,785	5,408
8	1,397	1,860	2,306	2,896	3,355	4,501	5,041
9	1,383	1,833	2,262	2,821	3,250	4,297	4,781
10	1,372	1,812	2,228	2,764	3,169	4,144	4,587
11	1,363	1,796	2,201	2,718	3,106	4,025	4,437
12	1,356	1,782	2,179	2,681	3,055	3,930	4,318
13	1,350	1,771	2,160	2,650	3,012	3,852	4,221
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15	1,341	1,753	2,131	2,602	2,947	3,733	4,073
16	1,337	1,746	2,120	2,583	2,921	3,686	4,015
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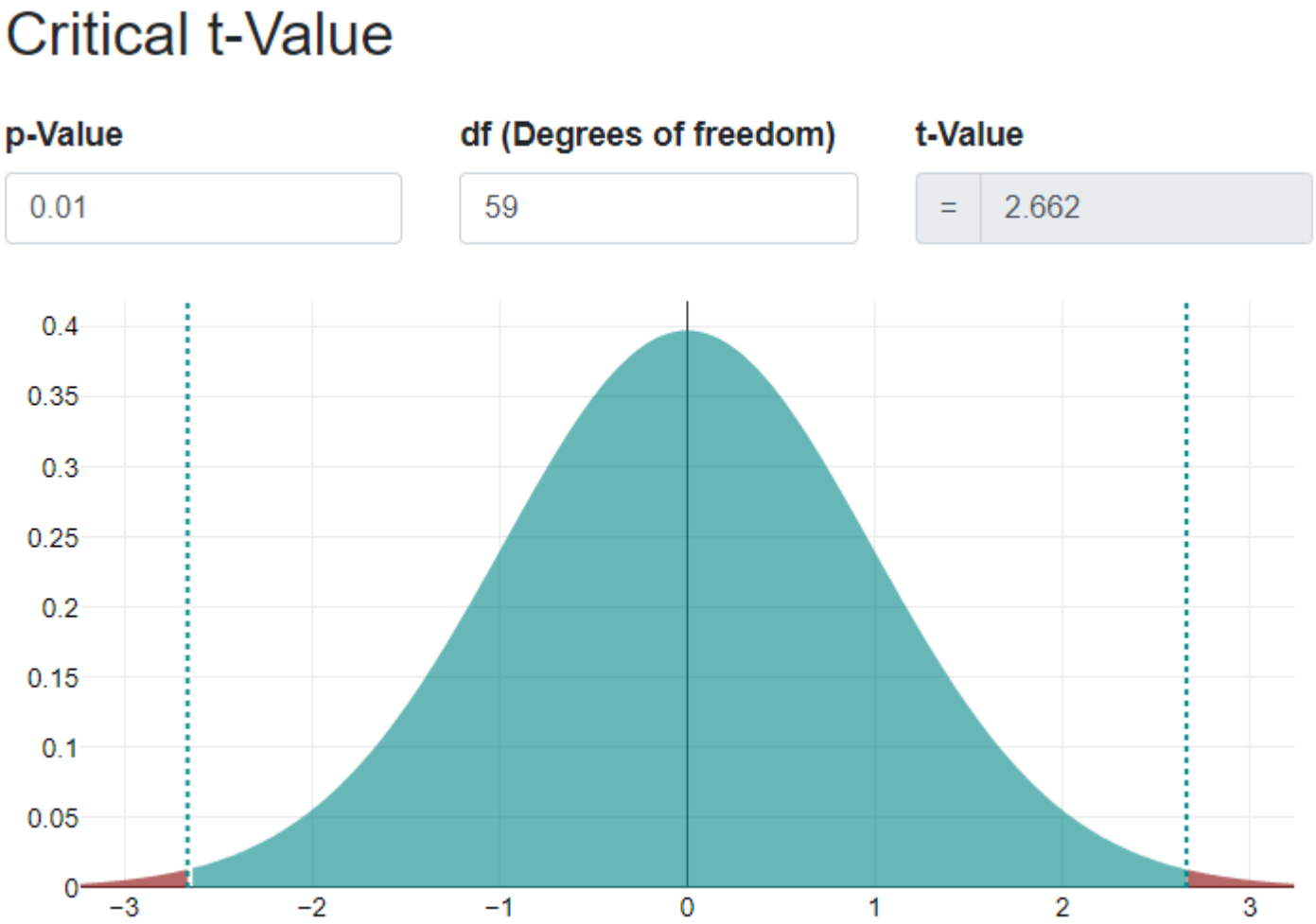
120	1,289	1,658	1,980	2,358	2,617	3,160	3,373
∞	1,282	1,645	1,960	2,326	2,576	3,090	3,291

Solution



https://datatab.net/tutorial/t-distribution

Statistical hypothesis testing

Task 2

In plant A, for 10 randomly selected employees, the mean age was 32 years and the standard deviation s = 4 years was obtained. Can we assume (assuming that the employees' ages are normally distributed) that the average age of an employee in this plant is significantly higher than 30 years? Significance level $\alpha = 0.05$.

Solution

Data:

- $n = 10$
- $m = 32$
- $s = 4$
- $\alpha = 0.05$

Hypotheses:

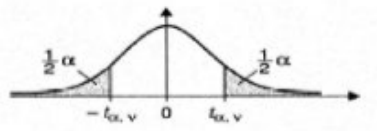
- $H_0: m_0 > 30$) *null hypothesis)*
- $H_1: m_0 < 30$) *alternative hypothesis)*
- One-sided test

Test Statistic Calculation:

$$t_e = \frac{32 - 30}{4} \cdot \sqrt{10} = 1.5$$

Answer: There are no grounds to reject hypothesis H_0.

***T-Student* distribution**



$P(|t| \geq t_{\alpha,\nu}) = \alpha$

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∞	1,282	1,645	1,960	2,326	2,576	3,090	3,291

Statistical hypothesis testing

Task 3

The operating time of a certain type of battery has a distribution N (m, 70). At the significance level $\alpha = 0.05$, test the hypothesis that the average operating time of this type of battery is over 500 h, if for 16 randomly selected batteries $\bar{X} = 560$ h is obtained.

Solution

Data:

- $n = 16$
- $m = 560$ h
- $s = 70$
- $\alpha = 0.05$

Hypotheses:

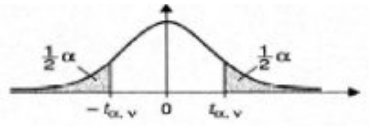
- $H_0: m_0 = 500$) null hypothesis)
- $H_1: m_0 \neq 500$) alternative hypothesis)
- two-sided test

Test Statistic Calculation:

$$t_e = \frac{560 - 500}{70} \cdot \sqrt{16} = 3.319$$

Answer: There are grounds to reject hypothesis H_0

***T-Student* distribution**



$P(|t| \geq t_{\alpha,\nu}) = \alpha$

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Statistical hypothesis testing

In randomly selected farms in the Mazovian and Wielkopolska voivodeships, the average sugar beet yields were examined. It is known that in both of these voivodeships, sugar beet yields have a normal distribution with a standard deviation of 20 q/ha. The mean from a sample of size $n_1 = 6$ drawn from the Mazovian voivodeship was 310 q/ha, while the mean from a sample of size $n_2 = 10$ drawn from the Wielkopolska voivodeship was 318 q/ha. Assuming a significance level $\alpha = 0.1$, verify the hypothesis that the average sugar beet yields obtained by farms in both voivodeships

Solution

Data:

- $n_1 = 6$
- $n_2 = 10$
- $\alpha = 0.1$
- $\bar{x}_1 = 310$ q/ha
- $\bar{x}_2 = 318$ q/ha
- $s_1 = 20$ q/ha
- $s_2 = 20$ q/ha

Hypotheses:

- $H_0: \mu_1 = \mu_2$) null hypothesis)
- $H_1: \mu_1 \neq \mu_2$) alternative hypothesis)

Model I:

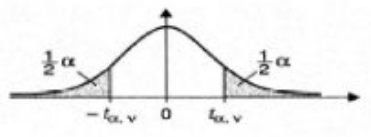
- $s = 6 + 10 - 2 = 14$
- $t_{\alpha,s} = 1.761$

Test Statistic Calculation:

$$u = \frac{310 - 318}{\sqrt{\frac{20^2}{6} + \frac{20^2}{10}}} = \frac{-8}{\sqrt{106.67}} = \frac{-8}{10.33} = -0.774$$

Answer: There are no grounds to reject hypothesis H_0 .

***T-Student* distribution**



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120	1,289	1,658	1,980	2,358	2,617	3,160	3,373

Statistical hypothesis testing

Task 5

Two random samples of bean grains of two species were taken and the grain lengths were measured. For species A, the following were obtained: n=450, $\bar{x}$=12.3 mm, s=1.8 mm, while for species B, the following were obtained: n=500, $\bar{x}$=11.9 mm, s=2.1 mm. At the significance level of α=0.05, verify the hypothesis that the mean grain lengths of both bean species are the same.

Solution

Data:

- $n_1 = 450$
- $\bar{x}_1 = 12.3$
- $s_1 = 1.8$
- $n_2 = 500$
- $\bar{x}_2 = 11.8$
- $s_2 = 2.1$
- $\alpha = 0.05$

Hypotheses:

- $H_0: \mu_1 = \mu_2$) null hypothesis)
- $H_1: \mu_1 \neq \mu_2$) alternative hypothesis)

From the normal distribution: $u = 1.64$

Test Statistic Calculation:

$$u = \frac{12.3 - 11.8}{\sqrt{\frac{(1.8)^2}{450} + \frac{(2.1)^2}{500}}} = 3.1746$$

Answer: There are no grounds to reject hypothesis H_0 .

Normal distribution

Tabela 1: Dystrybuanta standardowego rozkładu normalnego N(0,1)

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Example of how to use the table

Let $\alpha = 0.05$. Then $1 - \frac{\alpha}{2} = 0.975$ .We look for u_α that fulfills the condition $F(u_\alpha) = 0.975$ .Inside the table, we search for 0.975. Next, we read the "row name," which is 1.9, and the "column name," which is 0.06. Adding these numbers together, we get $u_\alpha = 1.96$.

	...	0.06	...
⋮		↑	
1.9	←	0.975	
⋮			

Statistical hypothesis testing

Task 6

The diameter of the cells of the comb built by the bees was measured in two hives. For 7 randomly selected cells from the first hive, the following results were obtained (in mm):

5.36, 5.20, 5.28, 5.16, 5.30, 5.08, 5.23

Similarly, for the second hive we obtained:

5.15, 5.04, 5.30, 5.22, 5.19, 5.24, 5.12

At the significance level of $\alpha=0.05$, verify the hypothesis that the mean lengths and diameters of cells in combs from two different hives are equal.

Solution

Hypotheses:

- $H_0: \bar{x}_1 = \bar{x}_2$) null hypothesis)
- $H_1: \bar{x}_1 \neq \bar{x}_2$) alternative hypothesis)

Parameters:

- $\alpha = 0.05$
- $n = 7 + 7 - 2 = 12$
- $u = 1.64$

	ul1	ul2
	5.36	5.15
	5.2	5.04
	5.28	5.3
	5.16	5.22
	5.3	5.19
	5.08	5.24
	5.23	5.12
mean	**5.23**	**5.18**
standard deviation	**0.09363**	**0.08544**

Test Statistic Calculation:

$$u = \frac{5.23 - 5.18}{\sqrt{\frac{(0.03)^2}{7} + \frac{(0.03)^2}{7}}} = 1.071$$

Answer: There are no grounds to reject hypothesis H_0 .

Normal distribution

Tabela 1: Dystrybuanta standardowego rozkładu normalnego N(0,1)

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Example of how to use the table

Let $\alpha = 0.05$. Then $1 - \frac{\alpha}{2} = 0.975$ .We look for u_α that fulfills the condition $F(u_\alpha) = 0.975$ .Inside the table, we search for 0.975. Next, we read the "row name," which is 1.9, and the "column name," which is 0.06. Adding these numbers together, we get $u_\alpha = 1.96$.

	...	0.06	...
⋮		↑	
1.9	←	0.975	
⋮			

Statistical hypothesis testing

Task 7

In order to test the hypothesis that the use of a different material increases the service life of a certain machine part, the service life of this part manufactured from the old and new material was tested on two samples. The following results were obtained for the old material A and the new material B (table). At the significance level of $\alpha=0.05$, verify the hypothesis that parts made of material B have a longer average service life.

Part lifetime (weeks)	Number of pieces	Material A	Material B
4-6		5	4
6-8		15	10
8-10		40	56
10-12		20	30
12-14		10	20

Solution

Data:

- $\alpha = 0.05$ (one-sided test)
- $u = 1.28$

Hypotheses:

- $H_0: \bar{x}_1 < \bar{x}_2$) null hypothesis)
- $H_1: \bar{x}_1 > \bar{x}_2$) alternative hypothesis)

Test Statistic Calculation:

$$u = \frac{18 - 24}{\sqrt{\frac{(13.5)^2}{5} + \frac{(20.4)^2}{5}}} = -0.548$$

Part lifetime (weeks)	Number of pieces	Material A	Material B
04.Jun		5	4
06.Aug		15	10
08.Oct		40	56
10.Dec		20	30
12-14		10	20
average		18	24
standard deviation		13.50926	20.44505

Answer: There are no grounds to reject hypothesis H_0 .

Normal distribution

Tabela 1: Dystrybuanta standardowego rozkładu normalnego N(0,1)

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Example of how to use the table

Let $\alpha = 0.05$. Then $1 - \frac{\alpha}{2} = 0.975$ .We look for u_α that fulfills the condition $F(u_\alpha) = 0.975$ .Inside the table, we search for 0.975. Next, we read the "row name," which is 1.9, and the "column name," which is 0.06. Adding these numbers together, we get $u_\alpha = 1.96$.

	...	0.06	...
⋮		↑	
1.9	←	0.975	
⋮			

Statistical hypothesis testing

Zad. 8

Dokonano 11 niezależnych pomiarów średnicy odlewanych rur i uzyskano następujące wyniki w mm: 50,2, 50,4, 50,6, 50,5, 49,9, 50,0, 50,3, 50,1, 50,0, 49,6, 50,6. Należy na poziomie istotności $\alpha = 0{,}05$ sprawdzić hipotezę, że wariancja uzyskiwanych średnic rur wynosi 0,04.

Solution

	50.2
	50.4
	50.6
	50.5
	49.9
	50
	50.3
	50.1
	50
	49.6
	50.6
standard deviation	0.316228

Data:

- $n = 11 - 1 = 10$
- $\alpha = 0.05$
- $S^2 = \left(0.316\right)^2$
- $\delta^2 = 0.04$

Hypotheses:

- $H_0: s^2 = \delta^2$) null hypothesis)
- $H_1: s^2 \neq \delta^2$) alternative hypothesis)

Test Statistic Calculation:

$$\chi^2 = \frac{n \cdot S^2}{\delta^2} = \frac{10 \cdot \left(0.316\right)^2}{0.04} = 24.86$$

n/α	0,995	0,99	0,98	0,975	0,97	0,96	0,95	0,9	0,1	0,05	0,04	0,03	0,025	0,02	0,01	0,005
1	0,0000	0,0002	0,0006	0,0010	0,0014	0,0025	0,0039	0,0158	2,7055	3,8415	4,2179	4,7093	5,0239	5,4119	6,6349	7,8794
2	0,0100	0,0201	0,0404	0,0506	0,0609	0,0816	0,1026	0,2107	4,6052	5,9915	6,4378	7,0131	7,3778	7,8240	9,2103	10,5966
3	0,0717	0,1148	0,1848	0,2158	0,2451	0,3002	0,3518	0,5844	6,2514	7,8147	8,3112	8,9473	9,3484	9,8374	11,3449	12,8382
4	0,2070	0,2971	0,4294	0,4844	0,5351	0,6271	0,7107	1,0636	7,7794	9,4877	10,0255	10,7119	11,1433	11,6678	13,2767	14,8603
5	0,4117	0,5543	0,7519	0,8312	0,9031	1,0313	1,1455	1,6103	9,2364	11,0705	11,6443	12,3746	12,8325	13,3882	15,0863	16,7496
6	0,6757	0,8721	1,1344	1,2373	1,3296	1,4924	1,6354	2,2041	10,6446	12,5916	13,1978	13,9676	14,4494	15,0332	16,8119	18,5476
7	0,9893	1,2390	1,5643	1,6899	1,8016	1,9971	2,1673	2,8331	12,0170	14,0671	14,7030	15,5091	16,0128	16,6224	18,4753	20,2777
8	1,3444	1,6465	2,0325	2,1797	2,3101	2,5366	2,7326	3,4895	13,3616	15,5073	16,1708	17,0105	17,5345	18,1682	20,0902	21,9550
9	1,7349	2,0879	2,5324	2,7004	2,8485	3,1047	3,3251	4,1682	14,6837	16,9190	17,6083	18,4796	19,0228	19,6790	21,6660	23,5894
10	2,1559	2,5582	3,0591	3,2470	3,4121	3,6965	3,9403	4,8652	15,9872	18,3070	19,0207	19,9219	20,4832	21,1608	23,2093	25,1882
11	2,6032	3,0535	3,6087	3,8157	3,9972	4,3087	4,5748	5,5778	17,2750	19,6751	20,4120	21,3416	21,9200	22,6179	24,7250	26,7568
12	3,0738	3,5706	4,1783	4,4038	4,6009	4,9385	5,2260	6,3038	18,5493	21,0261	21,7851	22,7418	23,3367	24,0540	26,2170	28,2995
13	3,5650	4,1069	4,7654	5,0088	5,2210	5,5838	5,8919	7,0415	19,8119	22,3620	23,1423	24,1249	24,7356	25,4715	27,6882	29,8195
14	4,0747	4,6604	5,3682	5,6287	5,8556	6,2426	6,5706	7,7895	21,0641	23,6848	24,4855	25,4931	26,1189	26,8728	29,1412	31,3193
15	4,6009	5,2293	5,9849	6,2621	6,5032	6,9137	7,2609	8,5468	22,3071	24,9958	25,8162	26,8479	27,4884	28,2595	30,5779	32,8013
16	5,1422	5,8122	6,6142	6,9077	7,1625	7,5958	7,9616	9,3122	23,5418	26,2962	27,1356	28,1907	28,8454	29,6332	31,9999	34,2672
17	5,6972	6,4078	7,2550	7,5642	7,8324	8,2878	8,6718	10,0852	24,7690	27,5871	28,4450	29,5227	30,1910	30,9950	33,4087	35,7185
18	6,2648	7,0149	7,9062	8,2307	8,5120	8,9889	9,3905	10,8649	25,9894	28,8693	29,7451	30,8447	31,5264	32,3462	34,8053	37,1565
19	6,8440	7,6327	8,5670	8,9065	9,2004	9,6983	10,1170	11,6509	27,2036	30,1435	31,0367	32,1577	32,8523	33,6874	36,1909	38,5823
20	7,4338	8,2604	9,2367	9,5908	9,8971	10,4154	10,8508	12,4426	28,4120	31,4104	32,3206	33,4624	34,1696	35,0196	37,5662	39,9968
21	8,0337	8,8972	9,9146	10,2829	10,6013	11,1395	11,5913	13,2396	29,6151	32,6706	33,5972	34,7593	35,4789	36,3434	38,9322	41,4011
22	8,6427	9,5425	10,6000	10,9823	11,3125	11,8703	12,3380	14,0415	30,8133	33,9244	34,8673	36,0492	36,7807	37,6595	40,2894	42,7957
23	9,2604	10,1957	11,2926	11,6886	12,0303	12,6072	13,0905	14,8480	32,0069	35,1725	36,1311	37,3323	38,0756	38,9683	41,6384	44,1813
24	9,8862	10,8564	11,9918	12,4012	12,7543	13,3498	13,8484	15,6587	33,1962	36,4150	37,3891	38,6093	39,3641	40,2704	42,9798	45,5585
25	10,5197	11,5240	12,6973	13,1197	13,4840	14,0978	14,6114	16,4734	34,3816	37,6525	38,6416	39,8804	40,6465	41,5661	44,3141	46,9279
26	11,1602	12,1981	13,4086	13,8439	14,2190	14,8509	15,3792	17,2919	35,5632	38,8851	39,8891	41,1460	41,9232	42,8558	45,6417	48,2899
27	11,8076	12,8785	14,1254	14,5734	14,9592	15,6087	16,1514	18,1139	36,7412	40,1133	41,1318	42,4066	43,1945	44,1400	46,9629	49,6449
28	12,4613	13,5647	14,8475	15,3079	15,7042	16,3711	16,9279	18,9392	37,9159	41,3371	42,3699	43,6622	44,4608	45,4188	48,2782	50,9934
29	13,1211	14,2565	15,5745	16,0471	16,4538	17,1377	17,7084	19,7677	39,0875	42,5570	43,6038	44,9132	45,7223	46,6927	49,5879	52,3356
30	13,7867	14,9535	16,3062	16,7908	17,2076	17,9083	18,4927	20,5992	40,2560	43,7730	44,8336	46,1599	46,9792	47,9618	50,8922	53,6720
40	20,7065	22,1643	23,8376	24,4330	24,9437	25,7989	26,5093	29,0505	51,8051	55,7585	56,9459	58,4278	59,3417	60,4361	63,6907	66,7660
50	27,9907	29,7067	31,6639	32,3574	32,9509	33,9426	34,7643	37,6886	63,1671	67,5048	68,8039	70,4230	71,4202	72,6133	76,1539	79,4900
60	35,5345	37,4849	39,6994	40,4817	41,1504	42,2656	43,1880	46,4589	74,3970	79,0819	80,4820	82,2251	83,2977	84,5799	88,3794	91,9517
70	43,2752	45,4417	47,8934	48,7576	49,4953	50,7243	51,7393	55,3289	85,5270	90,5312	92,0241	93,8813	95,0232	96,3875	100,4252	104,2149
80	51,1719	53,5401	56,2128	57,1532	57,9553	59,2902	60,3915	64,2778	96,5782	101,8795	103,4588	105,4221	106,6286	108,0693	112,3288	116,3211
90	59,1963	61,7541	64,6347	65,6466	66,5093	67,9437	69,1260	73,2911	107,5650	113,1453	114,8057	116,8688	118,1359	119,6485	124,1163	128,2989
100	67,3276	70,0649	73,1422	74,2219	75,1419	76,6705	77,9295	82,3581	118,4980	124,3421	126,0794	128,2367	129,5612	131,1417	135,8067	140,1695

Answer: There are no grounds to reject hypothesis H_0 .