AdTrans project , 2023-1-PL01-KA220-HED-000158917

ADTRANS
ADVANCED DATA ANALYSIS FOR TRANSPORTATION AND MOBILITY ACADEMIC TEACHERS

Parametric Significance Tests

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Co-funded by the European Union

Parametric Significance Tests

Statistical hypothesis testing encompasses the principles and methods of making specific assumptions about the parameters or distribution of statistical characteristics of the general population based on sample results.

A statistical hypothesis is any judgment about a general population (the distribution of a feature in the general population - nonparametric hypotheses or the values of its parameters - parametric hypotheses) issued without conducting a complete study, its truth is determined on the basis of a random sample.

Assumptions about the general population based on a sample are called statistical hypotheses .

Drawing conclusions about the validity of the presented judgments is called testing or verifying statistical hypotheses .

Parametric Significance Tests

The process of verifying a hypothesis follows a certain procedure called a statistical test .

Tests based on the results from a random sample allow a decision to be made about whether to reject or not to state a hypothesis.

Types of statistical hypotheses :

- *parametric* - refer to the values of the general population parameters, such as the mean value, variance or structure component,
- *non-parametric* - concern the distribution of a random variable in the general population (statistical feature, interdependence of features, randomness of the sample).

Statistical hypothesis testing

- Statistical hypotheses are formulated assumptions about the distribution of a population.
- Parametric hypotheses are used to specify the values of parameters in the distribution of a population.
- H_0 - null hypothesis (directly tested)
- H_1 - alternative hypothesis - competing with respect to H_0

Statistical hypothesis testing

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Statistical hypothesis testing

- **Statistical test** – used to VERIFY a statistical hypothesis; it is a rule of procedure that assigns to each possible random sample a decision to accept or reject the hypothesis being tested.

- **Significance tests** – in which, on the basis of a random sample, a decision is made only to reject the hypothesis being tested or to state that there are no grounds for rejecting it.

- **Significance level** – the maximum permissible probability of making a type I error (rejection of a true hypothesis) that the researcher is willing to accept.

Statistical hypothesis testing

Null hypothesis	Decision	
	Accept H_0	Reject H_0
Null hypothesis is **true**	correct decision	Type I error
Null hypothesis is **false**	Type II error	correct decision

Example

We accept $H_0: \mu = 3600$

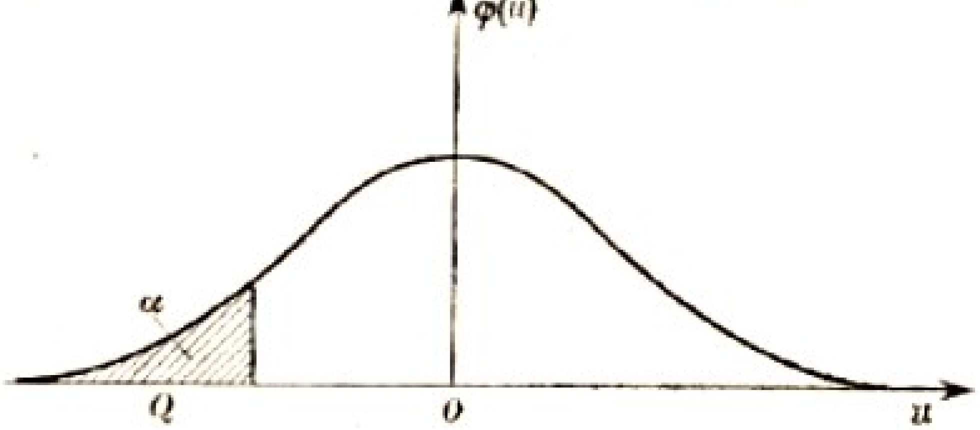
Then we can formulate H_1 as, for example:

- $H_1: \mu < 3600$) one-sided left-hand hypothesis)

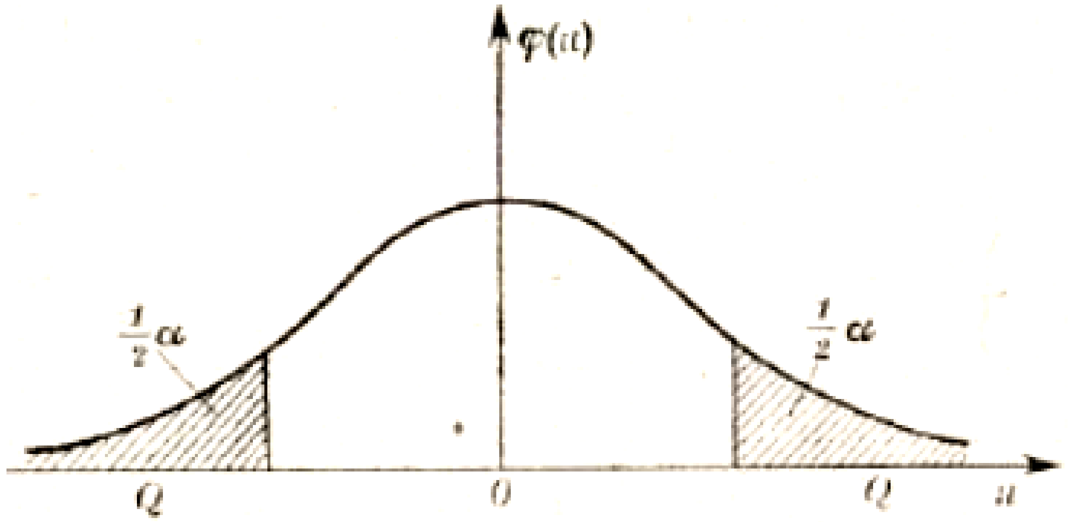
  or

- $H_1: \mu > 3600$) one-sided right-hand hypothesis)

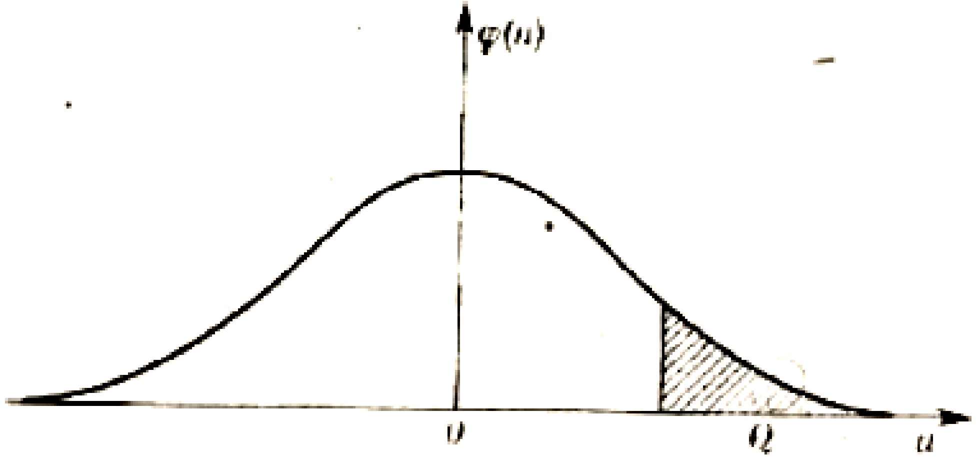
Critical area



Left-sided critical region Q



Two-sided critical region Q



Right-sided critical region Q

Sequence of procedure

- Formulation of the null and alternative hypotheses
- Accepting the level of significance
- Determination of the critical area (critical values)
- Selecting a test to verify the null hypothesis and calculating the value of the test function
- Making a verification decision

Significance tests for the mean

Model I	Model II	Model III

Significance tests for the mean

- Model I: The general population has a normal distribution $N(m, \sigma)$,with the standard deviation σ of the population being known. Based on the results of an n-element random sample, we need to check the hypothesis $H_0: m = m_0$) where m_0 is the hypothetical mean value) against the alternative hypothesis $H_1: m \neq m_0$.

$$u = \frac{\bar{x} - m_0}{\sigma}\sqrt{n}$$

- If the null hypothesis is true, the value of the statistic U calculated from the sample should not exceed the critical value u_α) determined from the N(0,1) distribution table).

$\pm u_{\alpha(\text{ for } 1-\frac{\alpha}{2})}$ -> **Two-sided critical region**
$u_{\alpha(\text{ for } 1-\alpha)}$ -> **Right-sided critical region**
$u_{\alpha(\text{ for } \alpha)}$ -> **Left-sided critical region**

Significance tests for the mean

Comparison of empirical and theoretical statistics:

a) Two-sided test: If $|u_{emp}| \geq u_{\alpha}$,we reject H_0 ;otherwise, there is no basis for rejection.

b) Right-sided test: If $u_{emp} \geq u_{\alpha}$,we reject H_0 ;otherwise, there is no basis for rejection.

c) Left-sided test: If $u_{emp} \leq u_{\alpha}$,we reject H_0 ;otherwise, there is no basis for rejection.

Significance tests for the mean

- Model II: The general population has a normal distribution $N(m, \sigma)$,but the standard deviation σ of the population is unknown. Based on the results of a small n-element random sample, the hypothesis $H_0\colon m = m_0$) where m_0 is the hypothetical mean value) should be tested against the alternative hypothesis $H_1\colon m \neq m_0$.

$$t = \frac{\bar{x} - m_0}{s}\sqrt{n-1}$$

- If the null hypothesis is true, the value of the statistic t calculated from the sample should not exceed the critical value t_α) obtained from Student's t-distribution table).

- In the case of a two-sided test: for $k = n - 1$ and significance level α,
- In the case of a one-sided test: for $k = n - 1$ and 2α.

Significance tests for the mean

Comparison of empirical and theoretical statistics:

a) Two-sided test: If $| t_{emp} | \geq t_{\alpha}$,we reject H_0 ;otherwise, there is no basis for rejection.

b) Right-sided test: If $t_{emp} \geq t_{\alpha}$,we reject H_0 ;otherwise, there is no basis for rejection.

c) Left-sided test: If $t_{emp} \leq -t_{\alpha}$,we reject H_0 ;otherwise, there is no basis for rejection.

Significance tests for the mean

- Model III: The general population has a normal distribution $N(m, \sigma)$ or any other distribution with mean value m and finite unknown variance σ^2 .Based on the results of a large n-element random sample (at least several dozen elements), the hypothesis $H_0: m = m_0$) where m_0 is the hypothetical mean value) should be tested against the alternative hypothesis $H_1: m \neq m_0$.

$$u = \frac{\bar{x} - m_0}{s}\sqrt{n}$$

- If the null hypothesis is true, the value of the statistic U calculated from the sample should not exceed the critical value u_α) determined from the N(0,1) distribution tables).

$\pm u_{\alpha(\text{ for } 1-\frac{\alpha}{2})}$ -> **Two-sided critical region**

$u_{\alpha(\text{ for } 1-\alpha)}$ -> **Right-sided critical region**

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Examples

7.1. The nominal weight of chocolate bars produced in a certain factory is 250 g. It is known that the distribution of weights of the produced chocolate bars is normal $N(m, 5)$ .For quality control purposes, a random sample of 16 chocolate bars was taken and the mean weight obtained was 244 g. Can we state that the chocolate production machine has gone out of adjustment and now produces chocolate bars with a weight less than the standard? At a significance level of $\alpha = 0.05$, verify the appropriate statistical hypothesis.

7.2. A sample of 26 patients was drawn, among whom the average blood pressure was 135, with a standard deviation of 45. At a significance level of 0.05, verify the hypothesis that these patients come from a population with an average blood pressure of 120.

Examples

7.3. The machine produces plates with a nominal thickness of 0.04 mm. A random sample of 25 plates gave an average thickness of 0.037 mm and a standard deviation of 0.005 mm. Can it be concluded that the produced plates are thinner than 0.04 mm? Use a significance level $\alpha = 0.01$.

7.4. In the study of employee absenteeism in an office, it was found that in a random sample of 100 employees of that office, the average duration of medical leave was 38 days, with a standard deviation of 16 days. Can it be concluded on this basis that the average annual leave time of the employees of this plant is longer than 31 days? Use a significance level $\alpha = 0.01$.

Significance tests for two means

Model I	Model II	Model III

Significance tests for two means

Model I: We study two general populations with normal distributions $N(m_1, \sigma_1)$ and $N(m_2, \sigma_2)$,where the standard deviations σ_1 and σ_2 of both populations are known. Based on the results of two independent samples of sizes n_1 and n_2 drawn from these populations, we need to test the hypothesis $H_0: m_1 = m_2$ against the alternative hypothesis $H_1: m_1 \neq m_2$.

$$u = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$\pm u_{\alpha(\text{ for } 1-\frac{\alpha}{2})}$ -> **Two-sided critical region**

$u_{\alpha(\text{ for } 1-\alpha)}$ -> **Right-sided critical region**

$u_{\alpha(\text{ for } \alpha)}$ -> **Left-sided critical region**

Significance tests for two means

Model II: We study two general populations with normal distributions $N(m_1, \sigma_1)$ and $N(m_2, \sigma_2)$,where the standard deviations σ_1 and σ_2 of both populations are unknown but equal ($\sigma_1 = \sigma_2$.(Based on the results of two independent small samples of sizes n_1 and n_2 drawn from these populations, we must test the hypothesis $H_0: m_1 = m_2$ against the alternative hypothesis $H_1: m_1 \neq m_2$.

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

The statistic follows Student's t-distribution with $n_1 + n_2 - 2$ degrees of freedom at the given significance level.

•In the case of a two-sided test: for $k = n_1 + n_2 - 2$ and significance level α,

•In the case of a one-sided test: for $k = n_1 + n_2 - 2$ and 2α.

Significance tests for two means

Model III: We study two general populations having normal distributions $N(m_1, \sigma_1)$ and $N(m_2, \sigma_2)$ or others, with finite unknown variances σ_1^2 and σ_2^2. Based on the results of two independent large samples (at least several dozen elements) of sizes n_1 and n_2 drawn from these populations, we need to test the hypothesis $H_0: m_1 = m_2$ against the alternative hypothesis $H_1: m_1 \neq m_2$.

$$u = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$\pm u_{\alpha(\text{ for } 1-\frac{\alpha}{2})}$ -> **Two-sided critical region**

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Example

8.1. We want to determine whether women employed in the same positions receive on average lower wages than men. From the population of women, an independent sample of $n_1 = 100$ was taken and the mean wage obtained was $\bar{x}_1 = 2180$ PLN and wage variance $s_1^2 = 6400$ .From the population of men employed in the same positions, an independent sample of $n_2 = 80$ was taken and the mean wage obtained was $\bar{x}_2 = 2280$ PLN and wage variance $s_2^2 = 10000$. At the significance level $\alpha = 0.01$, test the hypothesis that the average wages of women are lower.

8.2. It was hypothesized that the production time of a certain element can be reduced by using a different method. Under unchanged conditions, the production time using both methods was measured. For the new method, the following measurements were obtained: 15, 12, 10, 18, 14, 15, 13; for the old method: 17, 11, 22, 18, 19, 13, 14, 16. Verify the proposed hypothesis at the significance level $\alpha = 0.05$.

Significance test for variance

Model: The general population has a normal distribution $N(m, \sigma)$ with unknown parameters m and σ. Based on the results of an n-element random sample, the hypothesis $H_0 : \sigma^2 = \sigma_0^2$ should be tested against the alternative hypothesis $H_1 : \sigma^2 > \sigma_0^2$, where σ_0^2 is the value of the hypothetical variance.

$$\chi^2 = \frac{nS^2}{\sigma_0^2}$$

The obtained value is compared to the critical value χ_α^2) use χ^2 tables for $n - 1$ degrees of freedom and significance level α). If the calculated value is greater than the critical value at the given significance level, we reject the null hypothesis.

Example

9.1. Twelve voltage measurements were taken using a voltmeter, and from this sample, $s^2 = 0.9\,V^2$ was obtained. At the significance level $\alpha = 0.05$, test the hypothesis that the variance of the voltage measurements with this voltmeter is $0.6\,V^2$.

Significance test for fractions

Model: The general population has a binomial distribution with parameter p (i.e., the fraction of distinguished elements in the population is p).

From the population, a large, independently drawn sample of n elements ($n > 100$) was taken. Based on the results from this sample, the hypothesis $H_0 \colon p = p_0$ should be tested against the alternative hypothesis $H_1 \colon p \neq p_0$) where p_0 is the value of the hypothetical parameter p).

$$u = \frac{\frac{m}{n} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$$

$\pm u_{\alpha(\text{ for } 1-\frac{\alpha}{2})}$ -> **Two-sided critical region**

$u_{\alpha(\text{ for } 1-\alpha)}$ -> **Right-sided critical region**

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Example

9.3. It was hypothesized that the defect rate in the production of a certain element is 10%. To verify this hypothesis, an independent sample of 100 such elements was drawn, in which 15 defective elements were found. Verify the hypothesis at the significance level $\alpha = 0.05$.

9.4. In a production plant characterized by particularly high noise levels, an independent sample of $n = 160$ manual workers was drawn. Hearing tests revealed that 68 workers have disturbances in the ability to hear sounds with frequencies above 4000 vibrations/sec. At a significance level of 0.05, verify the hypothesis that more than 30% of the workers in this plant have these hearing disturbances.