AdTrans project , 2023-1-PL01-KA220-HED-000158917

ADTRANS
ADVANCED DATA ANALYSIS FOR TRANSPORTATION AND MOBILITY ACADEMIC TEACHERS

Operational Research - Mathematical Model
Formulation of Linear Programming Problems

Lukasz Jelinski

Co-funded by the European Union

Introduction

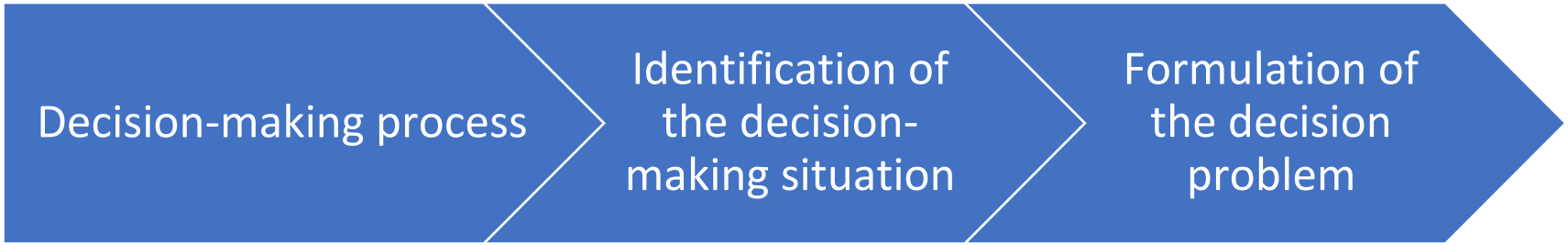
- **Operational research** - a scientific discipline related to decision theory that allows determining the method and solution of specific problems related to making optimal decisions.

- **Operational research** is a set of mathematical and statistical methods, including: mathematical programming, transportation problems, network algorithms, project management, inventory theory, queuing theory, Markov processes, time series analysis, gradient methods, neural networks, expert systems, and others.

Decision-making process

- **The decision-making process** or decision-making is a concept found in all areas of human activity, its basic components are selection criteria and alternatives.

- In classical decision theory, it means a group of logically related mental or computational operations leading to the solution of a decision problem by selecting one of the possible variants of action (decision).

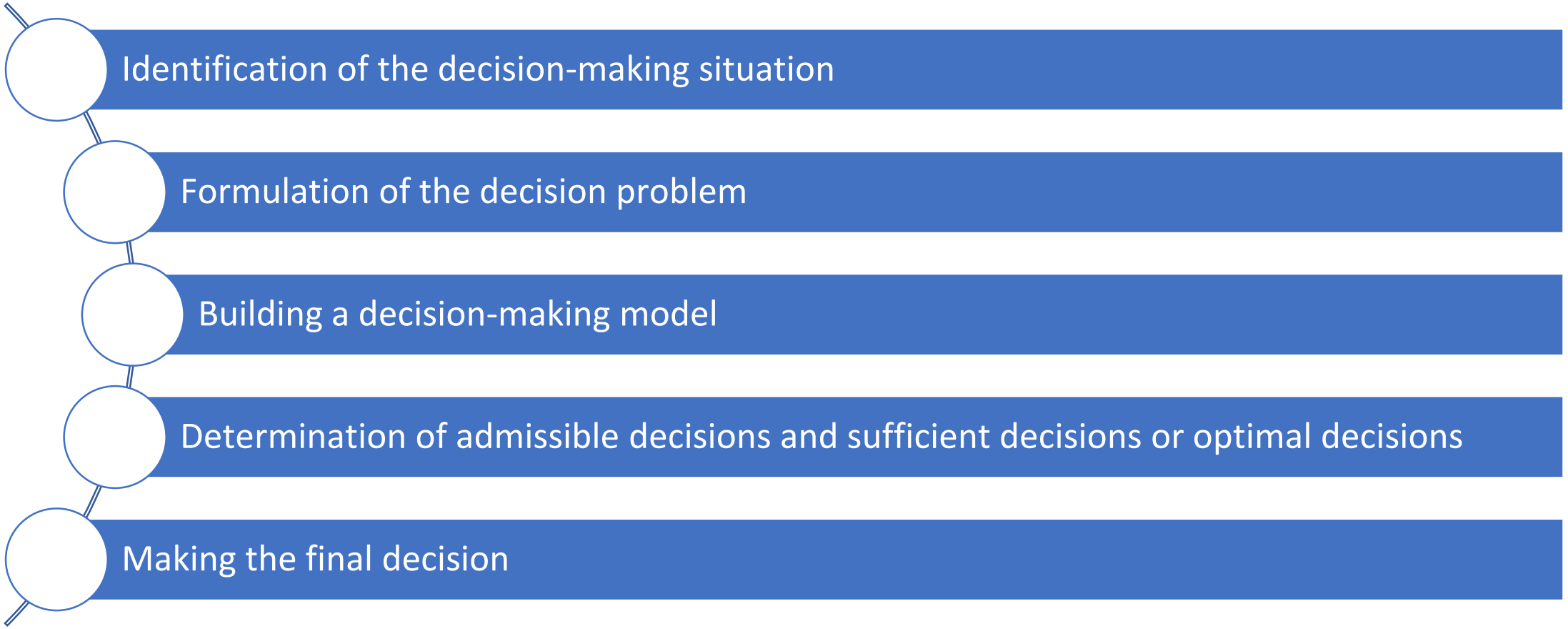


- **A decision-making problem** means a problematic situation in which the entity (decision-maker) is faced with the need to choose one of at least two possible variants of action.

Decision-making process

- **Formulating the decision problem** is usually the first step in building a decision model. A well-formulated problem should define in detail:
  - decision maker or decision makers
  - condition limiting the decision
  - a set of admissible decisions
  - decision evaluation criteria
- **A decision model** allows for a theoretical representation of a fragment of reality that synthetically describes the decision problem. Such a model should allow for determining **a set of admissible decisions** and **a set of optimal decisions** , if only such sets exist.
- **An admissible decision** is a decision that **satisfies all the limiting conditions** . The set of all such decisions is called **the set of admissible decisions** .

Decision-making process



Decision-making process

- **Decision** is a natural language concept, key in all sciences, and is the result of "making a decision" or, in other words, a decision-making process. The subtle difference between making a decision and a decision-making process is only that the decision-making process does not always lead to a decision.

- **A decision** can be an action/act or an opinion/judgment on a matter. For a decision-making process to make sense, at least two different choices (candidates for decisions) are needed, and therefore the existence of an alternative.

- We make decisions in many different situations. We call these situations decision situations , and the person making the decision a decision maker . The conditions in which the decision maker operates do not allow for the selection of any decision. A decision that complies with the constraining conditions is called an admissible decision .

Decision-making process

- Choosing the optimal decision requires adopting a specific criterion according to which we evaluate decisions as better or worse. This criterion is called the selection (evaluation) criterion.

- The description of a specific decision-making situation is called a decision problem (problem) in which the limiting conditions, selection criteria and decisions can be described in mathematical language.

- Constraint conditions are most often described by means of systems of equations or inequalities. These equations (or inequalities) will contain certain quantities (data), called parameters, and quantities that need to be determined, called decision variables.

Decision-making process

- **an admissible decision** with such a system of variable values (a system of numbers) that meet all the conditions describing the situation under study. The role of the selection criterion will be played by a certain function of decision variables measuring the goal that the decision-maker wants to achieve. This function is called the goal function .

- **The selection of the optimal decision** consists in determining such an admissible decision for which the objective function achieves the most advantageous value , i.e., depending on the situation under examination, the minimum or maximum value.

Mathematical model

- **Describing a decision-making situation** in mathematical language aims to reduce the problem of choosing the optimal decision to a solution of a certain, clearly defined mathematical task. In order for the solution of such a task to actually enable the selection of the best decision, it must be formulated in such a way that it precisely describes the given decision-making situation.

Building a linear programming model PL

- If in a decision problem the objective function and constraint conditions are linear, then such a task is called a linear optimization model .

- Linear optimization models (linear programming PL) constitute the largest and most widespread group of mathematical models related to the selection of the optimal decision.

Building a linear programming model PL

- A linear programming (LP) problem can have a feasible solution or be an inconsistent problem (no feasible solution). If a LP problem has a feasible solution, then one of three possibilities occurs:
  - there is one optimal solution,
  - there are many optimal solutions,
  - no optimal solution.

Formulation of the PL linear programming model

1.1. For the production of a certain medicine, two chemical compounds A and B are used. These compounds differ in chemical composition and purchase price.

Nutrient	Content of nutrient (mg) in 1 unit of compound		
	A	B	
Potassium (K)	0.2	0.5	
Sodium (Na)	0.7	1.0	
Nitrogen (N)	0.4	0.25	
Purchase price (PLN)	3	6	

It is known that the medicine must contain at least 2.6 mg of potassium, at least 7 mg of sodium, and up to 4 mg of nitrogen. In what quantities should chemical compounds A and B be purchased and mixed in order to prepare a medicine that meets the above requirements at the lowest possible cost? Formulate the linear programming problem.

Formulation of the PL linear programming model

1.2. The company produces two products: W_1 and W_2 .To produce them, two limited raw materials S_1 and S_2 are used. The consumption of these raw materials per unit of each product, permitted consumption limits, and unit profits from product sales are given in the table below.

PRODUCTS	CONSUMPTION PER UNIT OF PRODUCT		UNIT PROFIT (PLN)
	S1	S2	
W1	8	7	2
W2	16	4	4
Usage limit	96000	56000	

Additionally, it is known that the production capacity of one machine, which is the bottleneck of the production process, does not permit exceeding 5000 units of W_1 and 4000 units of W_2 .How many units of W_1 and how many units of W_2 should be produced, so as not to exceed the consumption limits, to maximize profit from product sales? Formulate the linear programming problem.

Formulation of the PL linear programming model

1.3. An amateur aquarist is considering buying food for fish. He is considering three brands: Rybex, Babyshark, and DXVC15. For the proper development of the hobbyist's pets, the content of certain nutrients designated as S1, S2, and S3 is important. The following table contains the nutrient content in each type of food and the price of one dekagram of each brand's food.

	S1	**S2**	**S3**	**PRICE [PLN]**
Rybex	5	0	3	5
Babyshark	1	2	4	4
DXVC15	3	1	5	3.5

The fish must be supplied with at least 10 units of S1 and at least 9 units of S3. The S2 content should not be more than 20 units, but also not less than 8 units. Develop a feed purchase plan so that the total purchase cost is minimal. Formulate the linear programming problem.

Sources:

- Materials from the subject posted on the eNauczanie website : Z. Kędra
- Z. Jędrzejczyk, J. Skrzypek, K. Kukuła, A. Walkosz : Operational research in examples and tasks. PWN. Warsaw, 1996
- M. Glinka: Elements of operational research in transport. Radom University of Technology Publishing House. Radom, 2009
- Other books and textbooks on Operational Research available in the PG Library