

Operational Research - Graphical method of solving linear programming problems

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Formulation of a linear programming model using the graphical method

The company produces two products: W1 and W2. The limitation in the production process is the working time of three machines: M1, M2, and M3. Table 1 shows the machine time used by each of these machines to produce a unit of each product, the allowable working times for each machine, and the prices of the products.

Machines	Machine time usage (hours) per unit		Allowable machine time (hours)	Available working time for the machine (hrs)
	W1	W2		
M1	2	1		1000
M2	3	3		2400
M3	1.5	-		600
Prices	30	20		

The task is to determine in what quantities to produce each product, so that with the existing limitations, the revenue from their sale is as high as possible. Will the optimal production structure change if the price of product W1 increases to 40 PLN?

Task 2.1

- ZD: W1, W2 $\Rightarrow x_1, x_2$
- FC: $30x_1 + 20x_2$ \square max
- WO:
 - $2x_1 + x_2 \leq 1000$
 - $3x_1 + 3x_2 \leq 2400$
 - $1.5x_1 \leq 600$
- WB: $x_1, x_2 \geq 0$

1. We have 2 decision variables \square we can solve graphically
2. For each WO equation we look for the points through which its graph passes.

$\triangleright x_1 x_2 = \square$

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$$\triangleright 2x_1 + x_2 = 1000 \square$$

$$\triangleright 3x_1 + 3x_2 = 2400 \square$$

$$\triangleright 1.5x_1 = 600 \square$$

$$x_1 = 0, x_2 = 1000$$

$$x_1 = 500, x_2 = 0$$

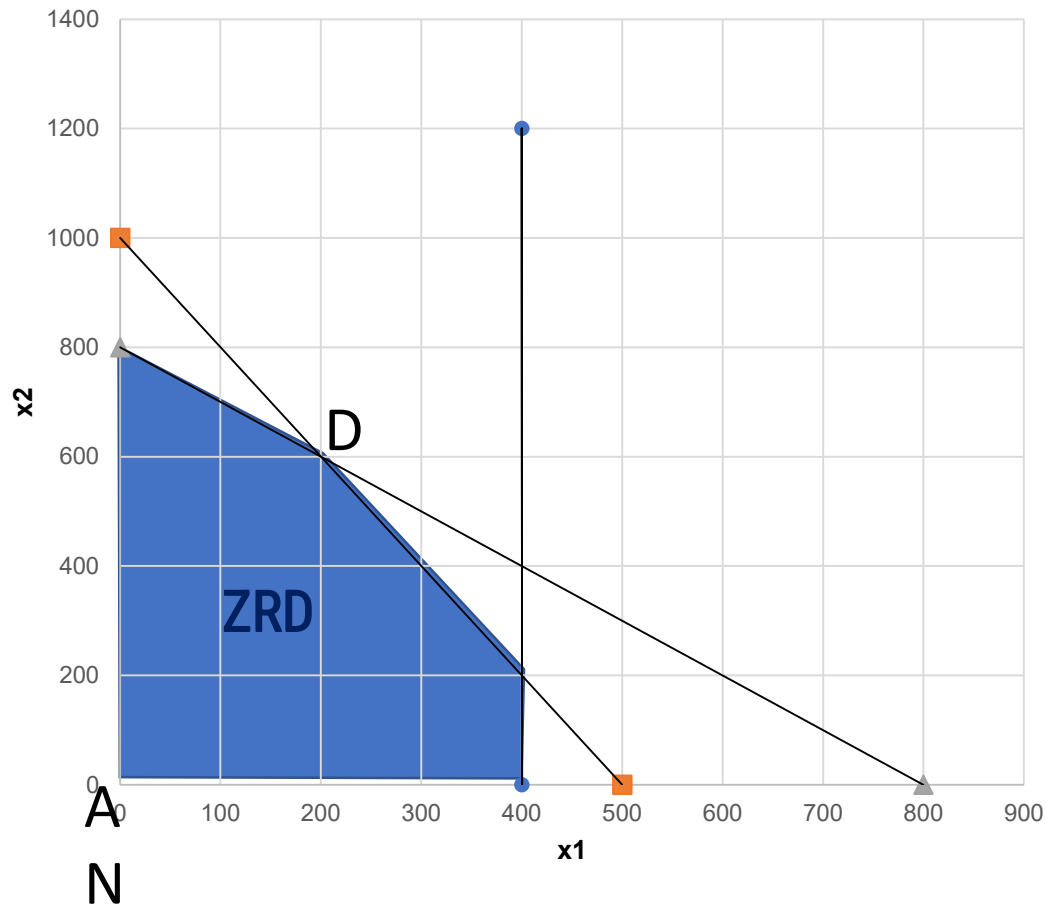
$$x_1 = 0, x_2 = 800$$

$$x_1 = 800, x_2 = 0$$

$$x_1 = 400$$

Task 2.1

3. We draw



4. We determine the ZRD (set of feasible solutions)

5. We look for the coordinates of the points and substitute them into FC

$$A(0,0) \quad F(A) = 0$$

$$B(400,0) \quad F(B) = 12000$$

$$C(400,200) \quad F(C) = (\text{intersection of 2 lines}) = 16000$$

$$D(200,600) \quad F(D) = (\text{intersection of 2 lines}) = 18000$$

$$E(0,800) \quad F(E) = 16000$$

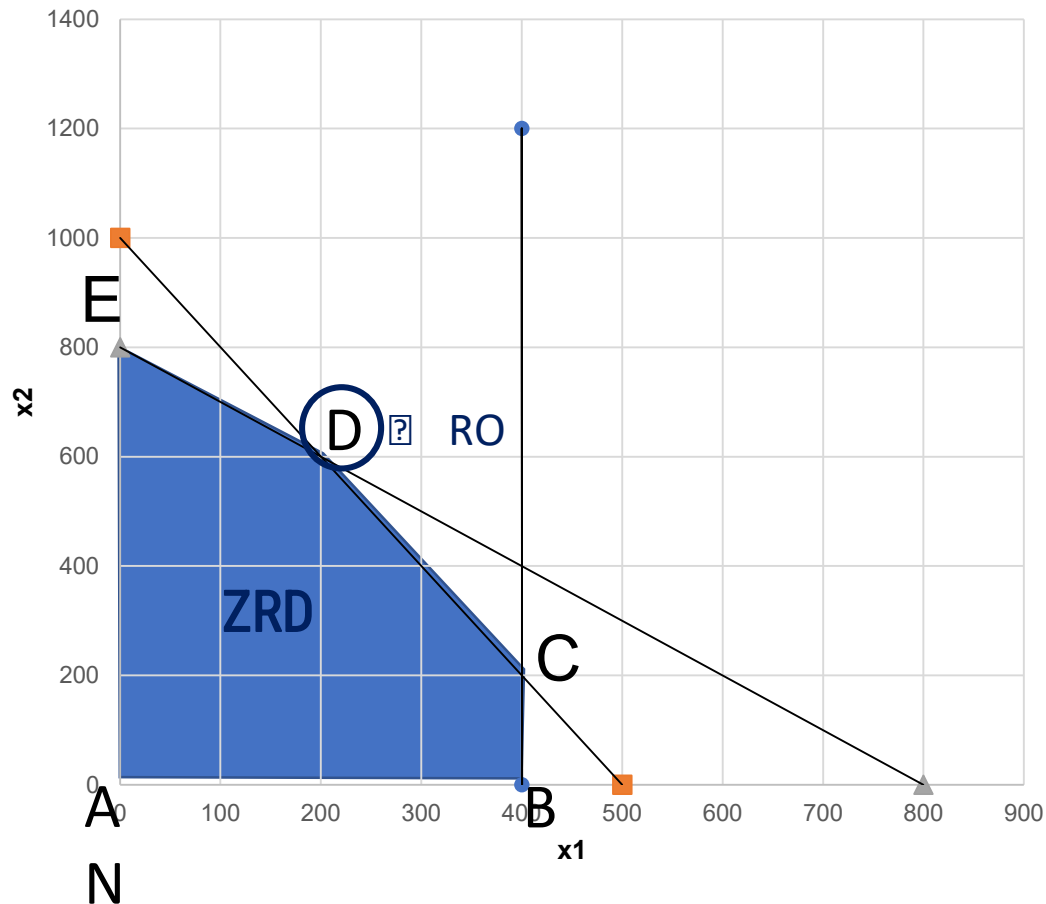
6. Optimal solution :

$$x_1 = 200 \quad x_2 = 600$$

$$FC = 18000$$

Task 2.1

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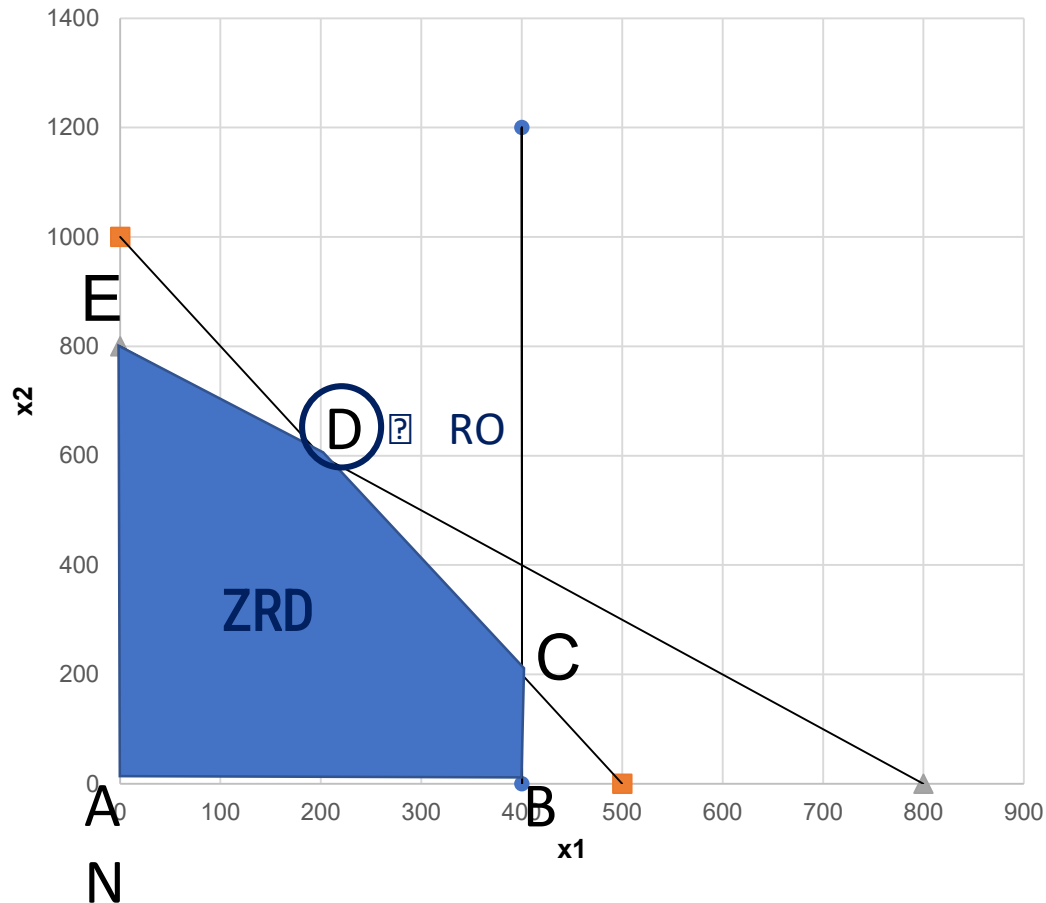
6. Optimal solution :

$$x_1 = 200 \quad x_2 = 600$$

$$FC = 18000$$

Task 2.1

3. We draw



8. Answer 1:

It is necessary to produce 200 units of W1 and 600 units of W2 to maximize the profit possible given the assumed constraints.

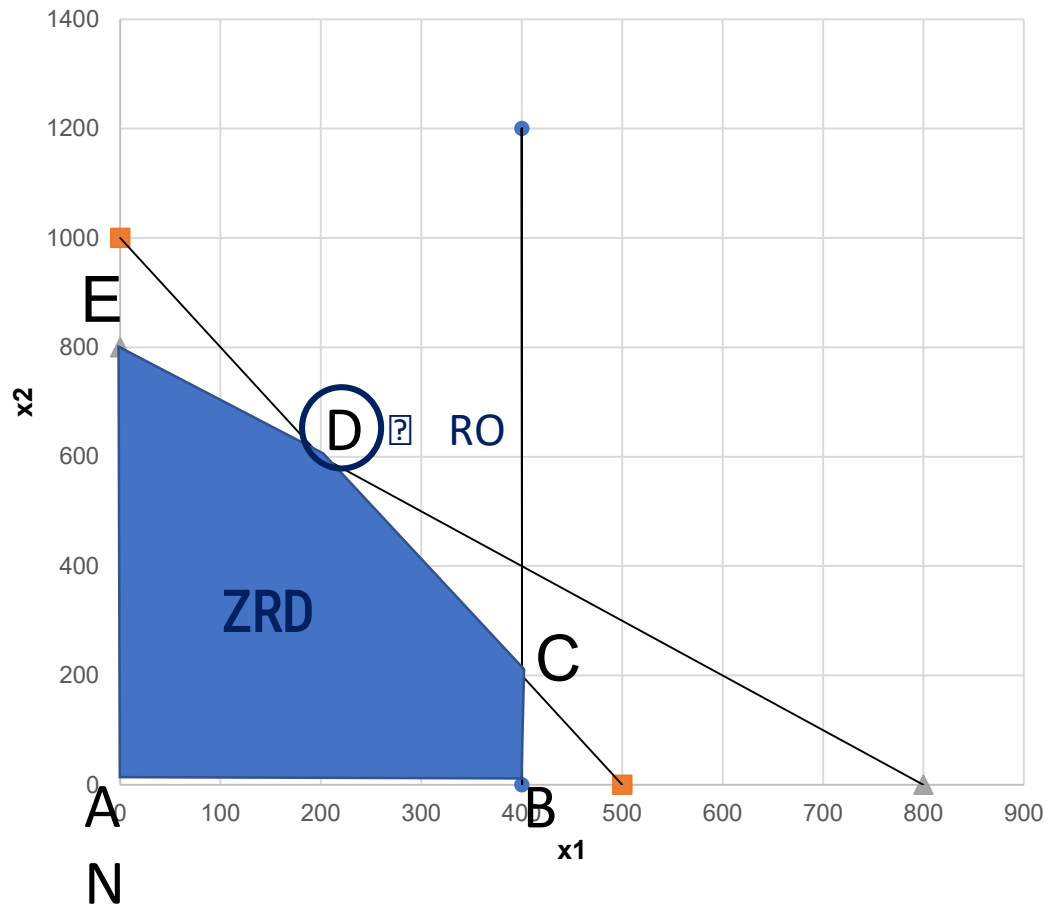
6. Optimal solution :

$$x_1 = 200, x_2 = 600$$

$$FC = 18000$$

Task 2.1

9. What if the price of W1 increases to PLN 40:



Will ZRD change?

ZRD will not change because nothing changes in WO

Will FC change?

FC will change to: $40x_1 + 20x_2$ max

Hence, the FC for point should be re-determined.

$$A(0,0) \quad F(A) = 0$$

$$B(400,0) \quad F(B) = 16,000$$

$$C(400,200) \quad F(C) = 20,000$$

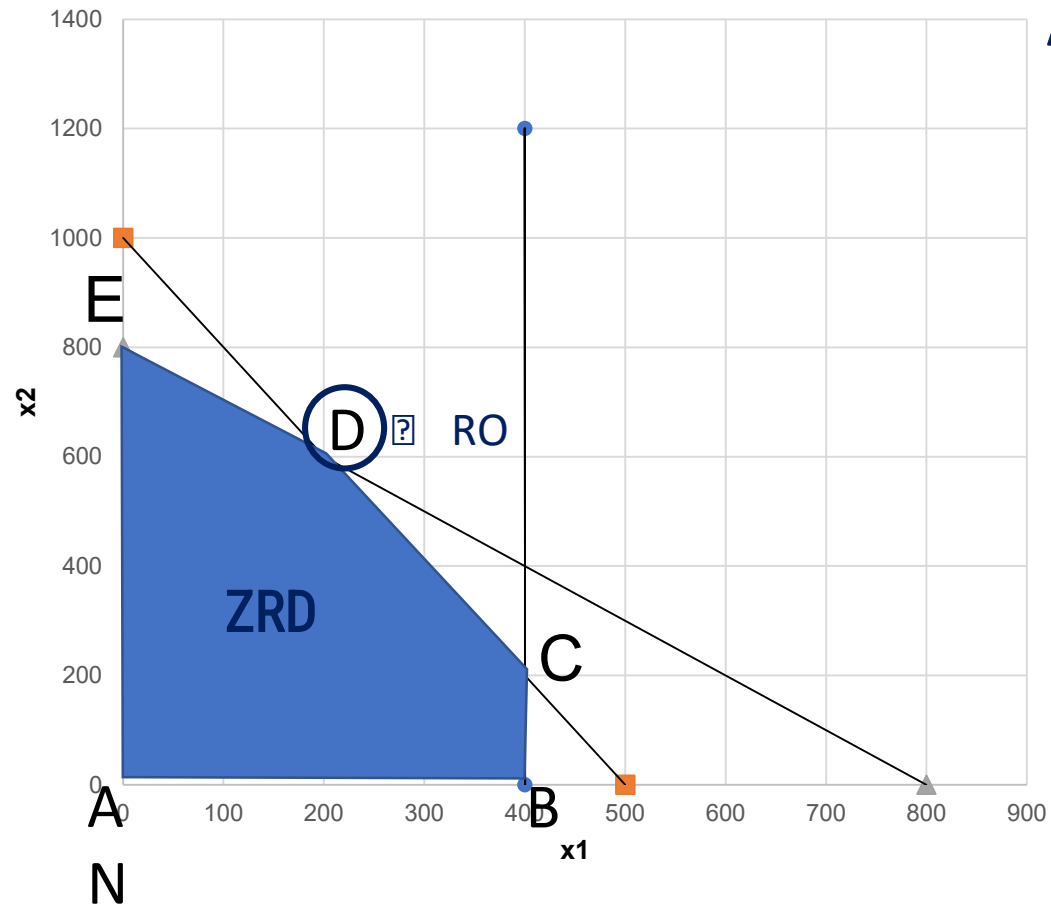
$$D(200,600) \quad F(D) = 20,000$$

$$E(0,800) \quad F(E) = 16,000$$

Task 2.1

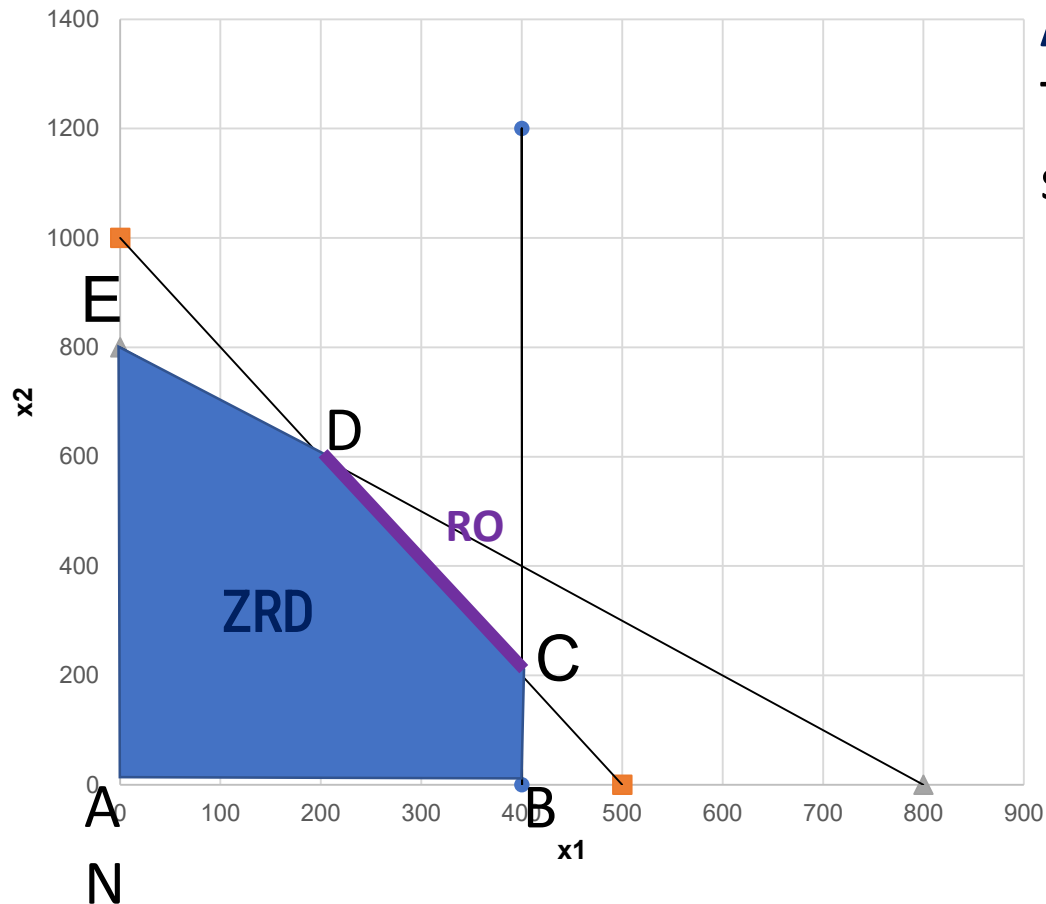
9. What if the price of W1 increases to PLN 40:

Application?



Task 2.1

9. What if the price of W1 increases to PLN 40:



Application?

The entire CD section will be a set of feasible solutions (how many optimal solutions)

Optimal solution:

$$x_1 = x_2 =$$

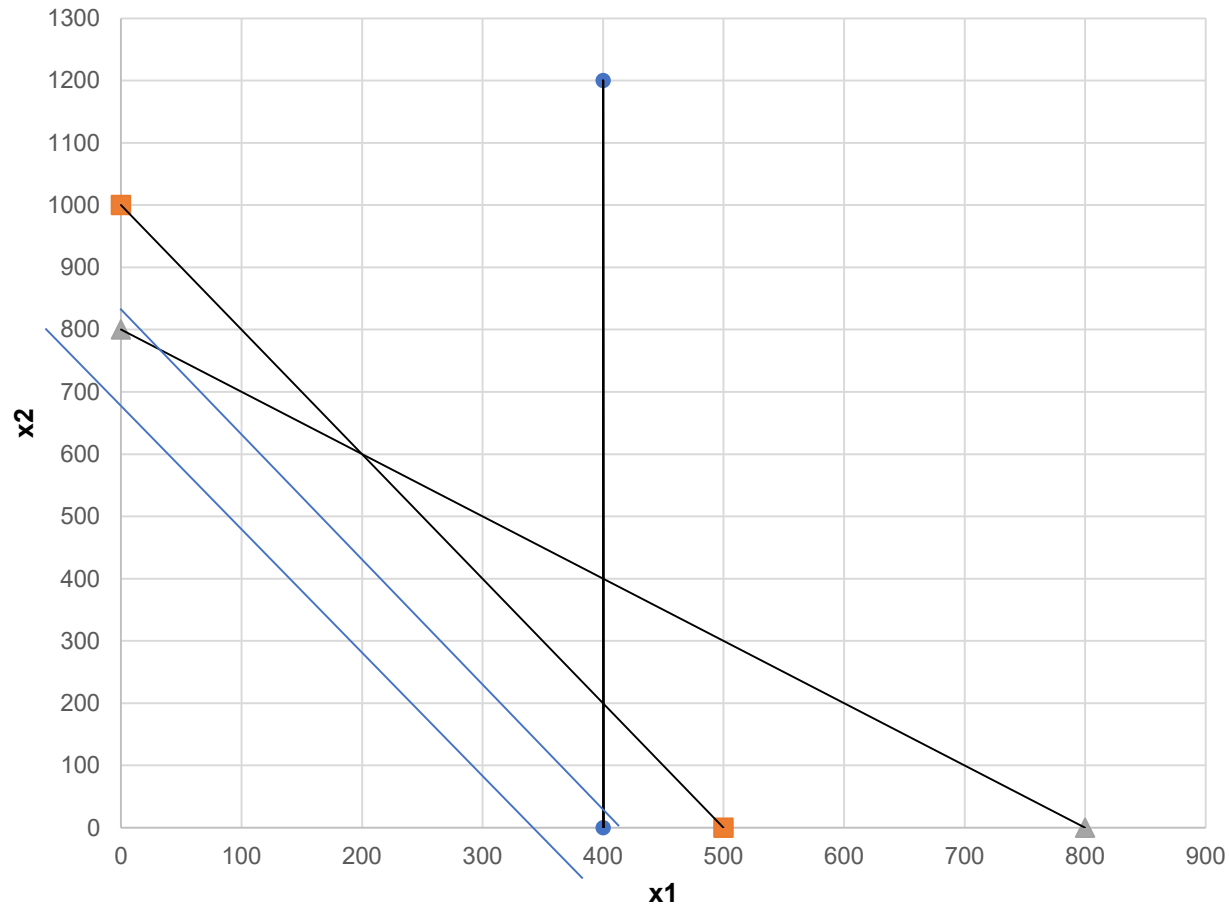
$$x_1 = x_2 =$$

$$FC =$$

$$FC =$$

10. Answer 2: The entire CD section will be a set of feasible solutions infinitely many optimal solutions

FC Isolines



How to determine the isolines of the Objective Function?
we substitute any number into FC, e.g.:

$$40x_1 + 20x_2 = 600$$

• $x_1 = x_2 =$

• $x_1 = x_2 =$

$$30x_1 + 20x_2 = 12000$$

• $x_1 = x_2 =$

• $x_1 = x_2 =$

Task 2.2

2.2. The following linear programming problem is given:

$$FC: 4x_1 + 2x_2 \rightarrow \max$$

$$WO: \begin{cases} 3x_1 + x_2 \leq 5 \\ 3x_1 - x_2 \geq -2 \end{cases}$$

$$WB: x_1, x_2 \geq 0$$

- Mark the set of feasible solutions together with the isolines of the objective function FC. Solve graphically and indicate the solution/set of optimal solutions.
- Formulate an additional constraint such that, when taken into account, the set of optimal solutions becomes infinite.
- Formulate another constraint such that, when taken into account, the linear programming problem will have no solution.



Task 2.2

2.2. The following linear programming problem is given:

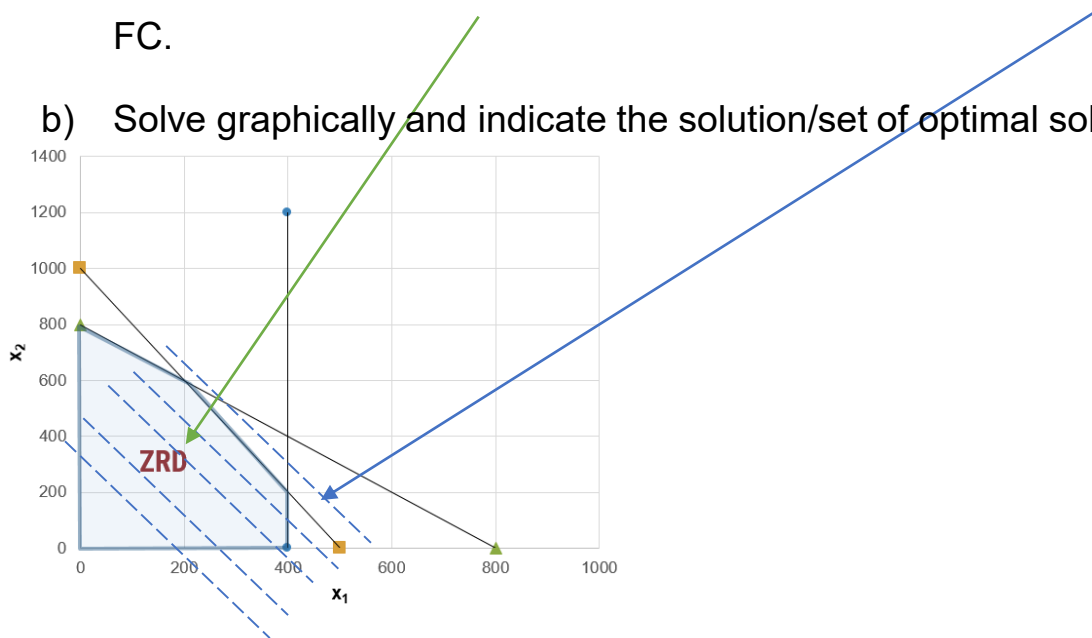
$$FC: 4x_1 + 2x_2 \rightarrow \max$$

$$WO: \begin{cases} 3x_1 + x_2 \leq 5 \\ 3x_1 - x_2 \geq -2 \end{cases}$$

$$WB: x_1, x_2 \geq 0$$

a) Mark the set of feasible solutions together with the isolines of the objective function FC.

b) Solve graphically and indicate the solution/set of optimal solutions.

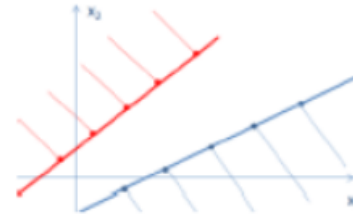




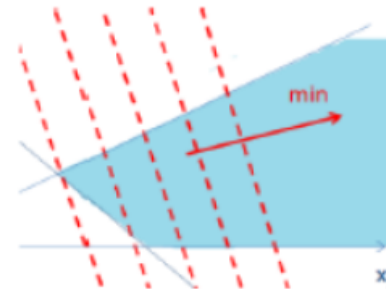
Task 2.2

Linear Programming Problem

Does not have a solution (contradictory LP, infeasible solution)
The feasible set (ZRD) is an empty set



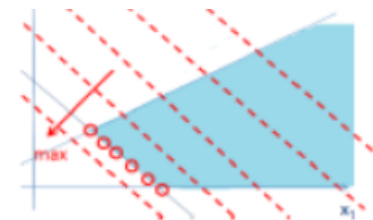
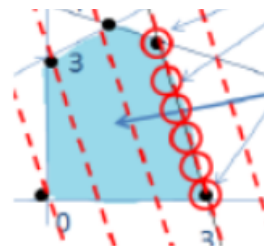
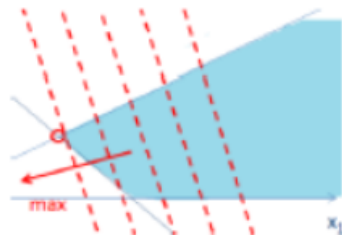
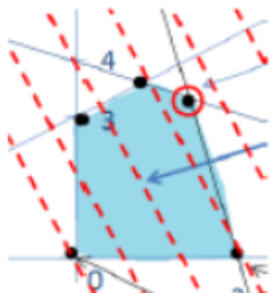
The objective function is unbounded (solution $+\infty / -\infty$)
The feasible set (ZRD) is an unbounded set



Has a bounded solution

Exactly one optimal solution

Infinitely many optimal solutions, for which the objective function (FC) assumes the same value



Task 2.2

Formulate an additional constraint so that, when it is included, the set of optimal solutions becomes infinite.

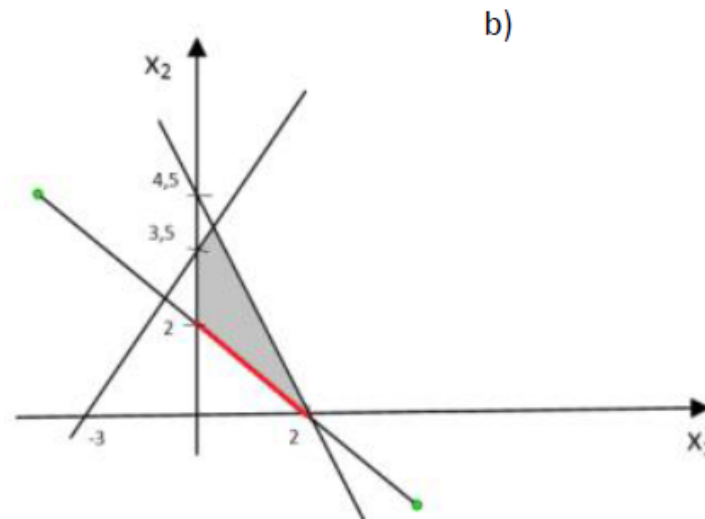
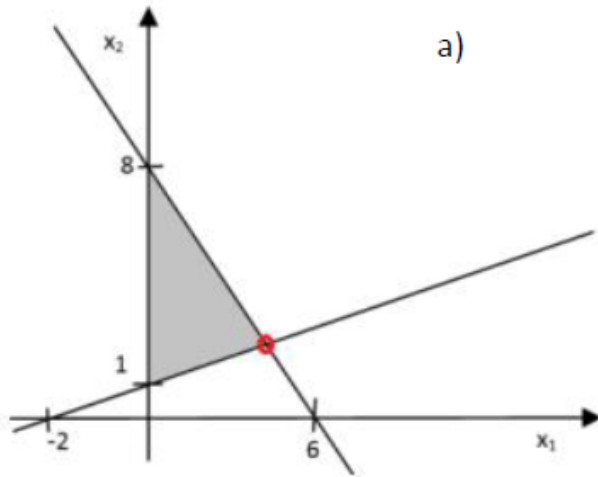
Infinite set of optimal solutions - additional WO, what?

Formulate another constraint so that, when it is included, the linear programming problem will have no solution.

Lack of optimal solutions - additional WO, what?

Task 2.5

2.5. Formulate a linear programming problem (description + model), whose solution by the graphical method is shown in the figure.



Give a solution. Formulate an additional constraint so that its inclusion causes the problem to have infinitely many optimal solutions (case a) and exactly one optimal solution (case b). Formulate another constraint so that its inclusion causes the linear programming problem to have no solution.

Task 2.8

2.8. Logs of length 5.6 m are cut at the sawmill into pieces of lengths 1.2, 1.6, and 1.9 m. The sawmill needs to execute a daily production plan that assumes delivering at least 200 logs of length 1.2 m, 300 logs of length 1.6 m, and 100 logs of length 1.9 m. How should the logs be cut to fulfill the plan while obtaining the least possible waste? As waste, consider pieces of wood shorter than 1.2 m. Construct a mathematical model of this problem.

Table

Cutting method/ Length of piece	I	II	III	IV	V	VI	VII	VIII	IX	Minutes plan
Waste:										-

Model:

ZD:
FC:
WO:
WB:

Sources:

- Materials from the subject posted on the eNauczanie website : Z. Kędra
- Z. Jędrzejczyk, J. Skrzypek, K. Kukuła, A. Walkosz : Operational research in examples and tasks. PWN. Warsaw, 1996
- M. Glinka: Elements of operational research in transport. Radom University of Technology Publishing House. Radom, 2009
- Other books and textbooks on Operational Research available in the PG Library