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ADTRANS
ADVANCED DATA ANALYSIS FOR TRANSPORTATION AND MOBILITY ACADEMIC TEACHERS

Operations Research – Simplex Algorithm for solving linear programming problems

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Solving linear programming problems using the simplex method

Solve the linear programming problem using the simplex method

$$FC: 30x_1 + 20x_2 \rightarrow max$$

$$WO: \begin{cases} 2x_1 + x_2 \leq 1000 \\ 3x_1 + 3x_2 \leq 2400 \\ 1{,}5x_1 \leq 600 \end{cases}$$

Task 3.1

Model

- FC: $_{30x1} + _{20x2}$? max
- WO:
  - $2x1 + _{x2} \leq 1000$
  - $3x_{1} + 3x_{2} \leq 2400$
  - $1.5x1 \leq _{600}$
- WB: $x_{1}, x_{2} \geq 0$

Base

Character

- $_{FC} : _{30x1} + _{20x2} + 0x3 + 0x4 + 0x_{5}$? max
- WO:
  - $2x1 + _{x2} + x3 = 1000$
  - $3x1 + _{3x2} + _{x4} = 2400$
  - $1.5x1 + x5 = 600$
- WB: $x_{1} : x_{5} \geq 0$

Task 3.1

Simplex board model: I board

c_j							b_j
c_b / x_b		x	x	x	x	x	
	x						
	x						
	x						
from $_j$							FC:
$c_j - z_j$							

- $_{FC}: 30x1 + 20x2 + 0x3 + 0x4 + 0x_5$ max
- WO:
  - $2x1 + _{x2} + x3 = 1000$
  - $3x1 + _{3x2} + _{x4} = 2400$
  - $1.5x1 + x5 = 600$
- WB: $x_1 : x_5 \geq 0$

Task 3.1

Simplex board model: I board

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
0	x3						
0	x4						
0	x5						
from $_j$							FC:
$c_j - z_j$							

- $FC: 30x1 + 20x2 + 0x3 + 0x4 + 0x5 \rightarrow max$
- WO:
  - $2x1 + x2 + x3 = 1000$
  - $3x1 + 3x2 + x4 = 2400$
  - $1.5x1 + x5 = 600$
- WB: $x1 : x5 \geq 0$

Task 3.1

Simplex board model: I board

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
0	x3	2	1	1	0	0	1000
0	x4	3	3	0	1	0	2400
0	x5	1.5	0	0	0	1	600
from $_j$							FC:
$c_j - z_j$							

- FC : $30x_1 + 20x_2 + 0x_3 + 0x_4 + 0x_5$ → max
- WO:
  - $2x_1 + x_2 + x_3 = 1000$
  - $3x_1 + 3x_2 + x_4 = 2400$
  - $1.5x_1 + x_5 = 600$
- WB: $x_1 : x_5 \geq 0$

Task 3.1

Simplex board model: I board

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
0	x3	2	1	1	0	0	1000
0	x4	3	3	0	1	0	2400
0	x5	1.5	0	0	0	1	600
from $_j$		0					FC:
$c_j - z_j$							

From $_1$
=0*2+0*3*0*1.5 = 0

Task 3.1

Simplex board model: I board

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
0	x3	2	1	1	0	0	1000
0	x4	3	3	0	1	0	2400
0	x5	1.5	0	0	0	1	600
from $_j$		0	0	0	0	0	FC:
$c_j - z_j$							

From $_1$
=0*2+0*3*0*1.5 = 0

Task 3.1

Simplex board model: I board

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
0	x3	2	1	1	0	0	1000
0	x4	3	3	0	1	0	2400
0	x5	1.5	0	0	0	1	600
from $_j$		0	0	0	0	0	FC:
$c_j - z_j$		30	20	0	0	0	0

We check the optimality criterion

In case of non-optimality, we exchange vectors in the simplex table base

Task 3.1

Simplex board model: I board

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
0	x3	2	1	1	0	0	1000
0	x4	3	3	0	1	0	2400
0	x5	1.5	0	0	0	1	600
from $_j$		0	0	0	0	0	FC:
$c_j - z_j$							0

Optimality criterion: (for maximization) the solution is optimal
when **the values of optimality indicators are non-positive (0 and -)**

We check the optimality criterion
In case of non-optimality, we exchange vectors in the simplex table base

Task 3.1

Simplex board model: I board

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
0	x3		1	1	0	0	1000
0	x4		3	0	1	0	2400
0	x5		0	0	0	1	600
from $_j$		0	0	0	0	0	FC:
$c_j - z_j$		30	20	0	0	0	0

MAX+

The base is entered	x
He leaves the base	x

Task 3.1

Simplex board model: I board

c_j			*20*	*0*	*0*	*0*	b_j
c_b / x_b			*x2*	*x3*	*x4*	*x5*	
0	*x3*		*1*	*1*	*0*	*0*	*1000*
0	*x4*		*3*	*0*	*1*	*0*	*2400*
$from_j$			*0*	*0*	*0*	*0*	FC:
$c_j - z_j$			*20*	*0*	*0*	*0*	*0*

MAX+

The base is entered	***x1***
He leaves the base	***x5***

Min quotient comes from the base

1000/2=500

2400/3=800

600/1.5=400 ⮜ min

Task 3.1

Simplex board model: II board

1,5	0	0	0	1	600

/(1.5)

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
0	x3	0					
0	x4	0					
30	x1	1					
from $_j$							
$c_j - z_j$							

Task 3.1

Simplex board model: II board

1,5	0	0	0	1	600

/(1.5)

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
0	x3	0					
0	x4	0					
30	x1	1	0	0	0	2/3	400
from $_j$							
$c_j - z_j$							

Task 3.1

Simplex board model: II board

		3	3	0	1	0	2400

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
0	x3	0					
0	x4	0	3				1200
30	x1	1	0	0	0	2/3	400
from $_j$							
$c_j - z_j$							

+

*(-3)

Task 3.1

Simplex board model: II board

		3	3	0	1	0	2400

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
0	x3	0					
0	x4	0	3	0	1	-2	1200
30	x1	1	0	0	0	2/3	400
from $_j$							
$c_j - z_j$							

+

*(-3)

Task 3.1

Simplex board model: II board

		2	1	1	0	0	1000

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
0	x3	0	1				
0	x4	0	3	0	1	-2	1200
30	x1	1	0	0	0	2/3	400
from $_j$							
$c_j - z_j$							

+

*(-2)

Task 3.1

Simplex board model: II board

		2	1	1	0	0	1000

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
0	x3	0	1	1	0	-4/3	200
0	x4	0	3	0	1	-2	1200
30	x1	1	0	0	0	2/3	400
from $_j$							
$c_j - z_j$							

+

*(-2)

Task 3.1

Simplex board model: II board

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
0	x3	0	1	1	0	-4/3	200
0	x4	0	3	0	1	-2	1200
30	x1	1	0	0	0	2/3	400
from $_j$		30					
$c_j - z_j$							

Task 3.1

Simplex board model: II board

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
0	x3	0	1	1	0	-4/3	200
0	x4	0	3	0	1	-2	1200
30	x1	1	0	0	0	2/3	400
from $_j$		30	0	0	0	20	1200
$c_j - z_j$							

Task 3.1

Simplex board model: II board

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
0	x3	0	1	1	0	-4/3	200
0	x4	0	3	0	1	-2	1200
30	x1	1	0	0	0	2/3	400
from $_j$		30	0	0	0	20	1200
$c_j - z_j$							

Task 3.1

Simplex board model: II board

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
0	x3	0	1	1	0	-4/3	200
0	x4	0	3	0	1	-2	1200
30	x1	1	0	0	0	2/3	400
from $_j$		30	0	0	0	20	1200
$c_j - z_j$		0	20	0	0	-20	

Task 3.1

Simplex board model: II board

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
0	x3	0	1	1	0	-4/3	200
0	x4	0	3	0	1	-2	1200
30	x1	1	0	0	0	2/3	400
from $_j$		30	0	0	0	20	12000
$c_j - z_j$							

Optimality criterion: (for maximization) the solution is optimal
when **the values of optimality indicators are non-positive (0 and -)**

We check the optimality criterion
In case of non-optimality, we exchange vectors in the simplex table base

Task 3.1

Simplex board model: II board

c_j		30		0	0	0	b_j
c_b / x_b		x1		x3	x4	x5	
0	x3	0		1	0	-4/3	200
0	x4	0		0	1	-2	1200
30	x1	1		0	0	2/3	400
from $_j$		30		0	0	20	12000
$c_j - z_j$		0		0	0	-20	

MAX+

Min quotient enters the base:

The base is entered	x
He leaves the base	x

Min quotient

Task 3.1

Simplex board model: II board

c_j		30		0	0	0	b_j
c_b / x_b		x1		x3	x4	x5	
0	x4	0		0	1	-2	1200
30	x1	1		0	0	2/3	400
from $_j$		30		0	0	20	12000
$c_j - z_j$		0		0	0	-20	

MAX+

Min quotient enters the base:

200/1= 200 ⍰ min Min quotient

1200/3=400

~~400/0~~

The base is entered	x2
He leaves the base	x3

Task 3.1

Simplex board model: III board

		0	1	1	0	-4/3	200

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
20	x2		1				
0	x4		0				
30	x1		0				
from $_j$							
$c_j - z_j$							

Task 3.1

Simplex board model: III board

		0	1	1	0	-4/3	200

/(1)

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
20	x2	0	1	1	0	-4/3	200
0	x4		0				
30	x1		0				
from $_j$							
$c_j - z_j$							

Task 3.1

Simplex board model: III board

		0	3	0	1	-2	1200

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
20	x2	0	1	1	0	-4/3	200
0	x4	0	0	-3	1	2	600
30	x1		0				
from $_j$							
$c_j - z_j$							

+

*(x)

Task 3.1

Simplex board model: III board

		0	3	0	1	-2	1200

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
20	x2	0	1	1	0	-4/3	200
0	x4	0	0	-3	1	2	600
30	x1		0				
from $_j$							
$c_j - z_j$							

+

*(-3)

Task 3.1

Simplex board model: III board

		1	0	0	0	2/3	400

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
20	x2	0	1	1	0	-4/3	200
0	x4	0	0	-3	1	2	600
30	x1		0				
$from_j$							
$c_j - z_j$							

+

*(x)

Task 3.1

Simplex board model: III board

		1	0	0	0	2/3	400

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
20	x2	0	1	1	0	-4/3	200
0	x4	0	0	-3	1	2	600
30	x1	1	0	0	0	2/3	400
from $_j$							
$c_j - z_j$							

+

*(0)

Task 3.1

Simplex board model: III board

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
20	x2	0	1	1	0	-4/3	200
0	x4	0	0	-3	1	2	600
30	x1	1	0	0	0	2/3	400
from $_j$		30	20	20	0	-20/3	16000
$c_j - z_j$							

Optimality criterion: (for maximization) the solution is optimal
when **the values of optimality indicators are non-positive (0 and -)**

We check the optimality criterion
In case of non-optimality, we exchange vectors in the simplex table base

Task 3.1

Simplex board model: III board

c_j		30	20	0	0		b_j
c_b / x_b		x1	x2	x3	x4		
20	x2	0	1	1	0		200
0	x4	0	0	-3	1		600
30	x1	1	0	0	0		400
from $_j$		30	20	20	0		16000
$c_j - z_j$		0	0	-20	0		

MAX+

Min quotient enters the base:

The base is entered	x
He leaves the base	x

Min quotient

Task 3.1

Simplex board model: III board

c_j		30	20	0	0		b_j
c_b / x_b		x1	x2	x3	x4		
20	x2	0	1	1	0		200
30	x1	1	0	0	0		400
from $_j$		30	20	20	0		16000
$c_j - z_j$		0	0	-20	0		

MAX+

Min quotient enters the base:

200/(-4/3)=-150

600/2=300 ⊠ min non-negative Min quotient

400/ 2/3 = 600

The base is entered	x5
He leaves the base	x4

Task 3.1

Simplex board model: IV board

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
20	x2					0	
0	x5					1	
30	x1					0	
from $_j$							
$c_j - z_j$							

Task 3.1

Simplex board model: IV board

		0	0	-3	1	2	600

/(2)

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
20	x2					0	
0	x5					1	
30	x1					0	
from $_j$							
$c_j - z_j$							

Task 3.1

Simplex board model IV board

0	1	1	0	-4/3	200
0	0	-3	1	2	600

/(2)

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
20	x2					0	
0	x5	0	0	-3/2	½	1	300
30	x1					0	
from $_j$							
$c_j - z_j$							

Task 3.1

Simplex board model: IV board

		0	1	1	0	-4/3	200

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
20	x2					0	
0	x5	0	0	-3/2	½	1	300
30	x1					0	
$from_j$							
$c_j - z_j$							

+

*(4/3)

Task 3.1

Simplex board model: IV board

		0	1	1	0	-4/3	200

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
20	x2	0	1	-1	2/3	0	600
0	x5	0	0	-3/2	½	1	300
30	x1					0	
from $_j$							
$c_j - z_j$							

+

*(4/3)

Task 3.1

Simplex board model: IV board

		1	0	0	0	2/3	400

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
20	x2	0	1	-1	2/3	0	600
0	x5	0	0	-3/2	½	1	300
30	x1					0	
from $_j$							
$c_j - z_j$							

+

*(-2/3)

Task 3.1

Simplex board model: IV board

		1	0	0	0	2/3	400

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
20	x2	0	1	-1	2/3	0	600
0	x5	0	0	-3/2	½	1	300
30	x1	1	0	1	-1/3	0	200
from $_j$							
$c_j - z_j$							

+

*(-2/3)

Task 3.1

Simplex board model: IV board

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
20	x2	0	1	-1	2/3	0	600
0	x5	0	0	-3/2	½	1	300
30	x1	1	0	1	-1/3	0	200
from $_j$		30	20	10	10/3	0	18000
$c_j - z_j$							

Task 3.1

Simplex board model: IV board

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
20	x2	0	1	-1	2/3	0	600
0	x5	0	0	-3/2	½	1	300
30	x1	1	0	1	-1/3	0	200
from $_j$		30	20	10	10/3	0	18000
$c_j - z_j$							

Optimality criterion: (for maximization) the solution is optimal
when **the values of optimality indicators are non-positive (0 and -)**

We check the optimality criterion
In case of non-optimality, we exchange vectors in the simplex table base

Task 3.1

Simplex board model: IV board

c_j		30	20	0	0	0	b_j
c_b / x_b		x1	x2	x3	x4	x5	
20		0	1	-1	2/3	0	
0	x5	0	0	-3/2	½	1	300
30		1	0	1	-1/3	0	
from $_j$		30	20	10	10/3	0	
$c_j - z_j$							

Optimal solution:

x1 = 200

x2 = 600

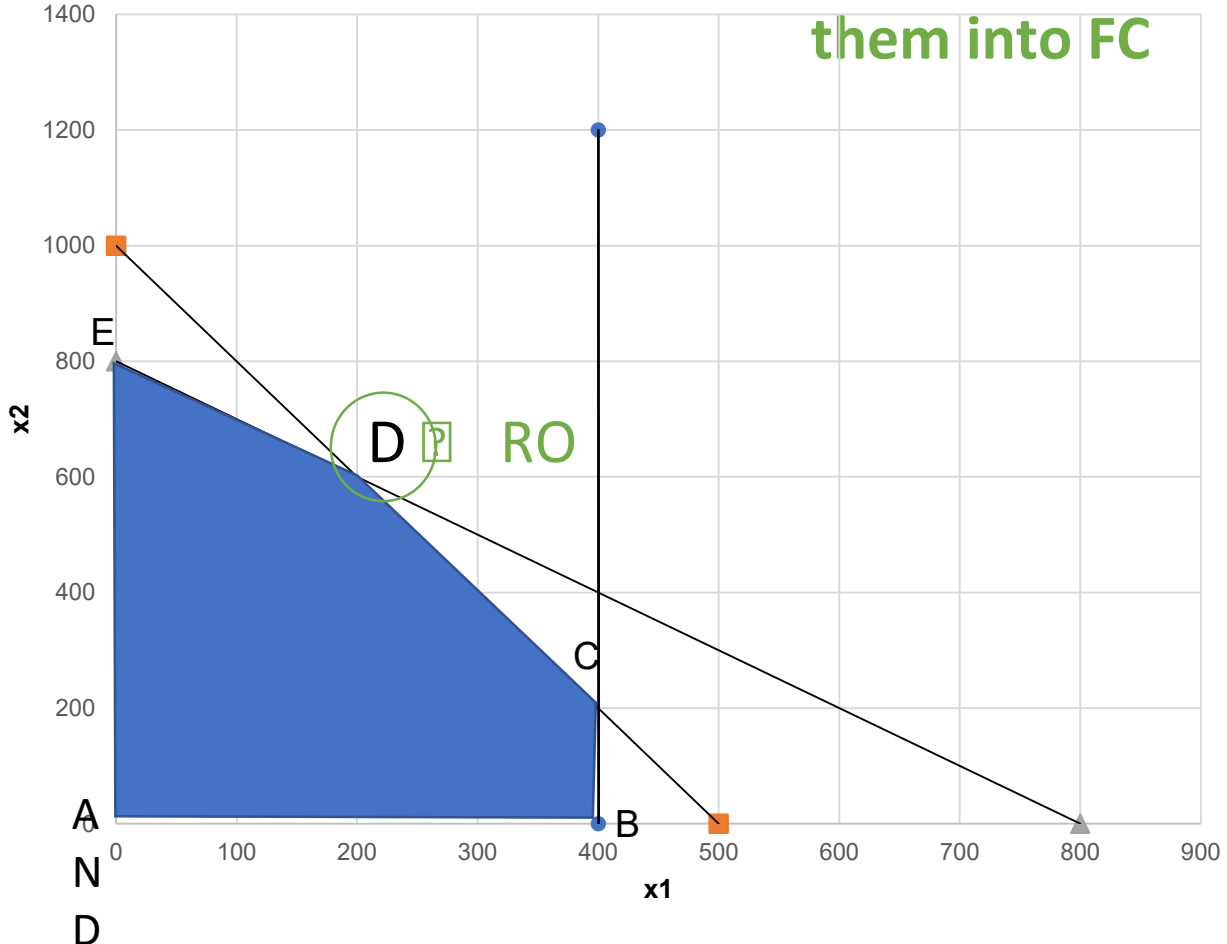
FC = 18000

Optimality criterion: (for maximization) the solution is optimal
when **the values of optimality indicators are non-positive (0 and -)**

We check the optimality criterion
In case of non-optimality, we exchange vectors in the simplex table base

Task 2.1 (graphic method)

3. We draw



4. We determine the ZRD (set of feasible solutions)

5. We look for the coordinates of the points and substitute them into FC

A(0,0) F(A) = 0

B(400,0) F(B)=1200

C(400,200) F(C)= (intersection of 2 lines) =1600

D(200,600) F(D)=1800

E(0,800) F(E)= (intersection of 2 lines) =1600

6. Optimal solution:

$x1 = 200x2 = 600$

FC =18000

Task 3.2

3.2. Rozwiąż zagadnienie programowania liniowego metodą simpleks

$$FC: 2x_1 + 3x_2 \rightarrow min$$

$$WO: \begin{cases} 3x_1 + 2x_2 \geq 6 \\ x_1 \leq 7 \end{cases}$$

Task 3.2

Model

- FC: $_{2x1}$ + $_{3x2}$ □ min
- WO:
  - 3x1 + $_{2x2}$ ≥6
  - x $_1$ ≤ 7
- WB: x $_1$, x $_2$ ≥ 0

Base Character

- FC: 2x $_1$ +3x $_2$ + … □ min
- WO:
  - 3x1 + $_{2x2}$ … = 6
  - x1 … = 7
- WB: x $_1$: x $_4$ ≥ 0

Task 3.2

Model

- FC: $_{2x1} + _{3x2}$? min
- WO:
  - $3x1 + _{2x2} \geq 6$
  - $x_1 \leq 7$
- WB: $x_1, x_2 \geq 0$

Base Character

- FC $_{: 2x1} + _{3x2} + 0x3 + 0x4 + Ms_1$? min
- WO:
  - $3x1 + _{2x2}$ -x3 +s $_1$ = 6
  - x1 + x4 $_=$ 7
- WB: $x_1 : x_4 \geq 0$

Task 3.2

Simplex board model: I board

c_j						b_j
c_b / x_b						
	x4					
from $_j$						FC:
$c_j - z_j$						

- FC $_{: 2x1}$ + $_{3x2}$ + 0x3 + 0x4 + Ms $_1$ → min
- WO:
  - 3x1 + $_{2x2}$ -x3 +s $_1$ = 6
  - x1 + x4 = 7
- WB: x $_1$: x $_4$ ≥ 0

Task 3.2

Simplex board model: I board

c_j		2	3	0	0	M	b_j
c_b / x_b		***x1***	***x2***	***x3***	***x4***	***s_1***	
M	***s_1***	***3***	***2***	***-1***	***0***	***1***	***6***
0	***x4***	***1***	***0***	***0***	***1***	***0***	***7***
from $_j$		***3M***	***2M***	***-M***	***0***	***M***	FC:
$c_j - z_j$		***2-3M***	***3-2M***	***M***	***0***	***0***	***6M***

- FC $_{:2x1}$ + $_{3x2}$ + 0x3 + 0x4 + Ms $_1$ → min
- WO:
  - 3x1 + $_{2x2}$ -x3 +s_1 = 6
  - x1 + x4 = 7
- WB: x_1 : x_4 ≥ 0

Task 3.2

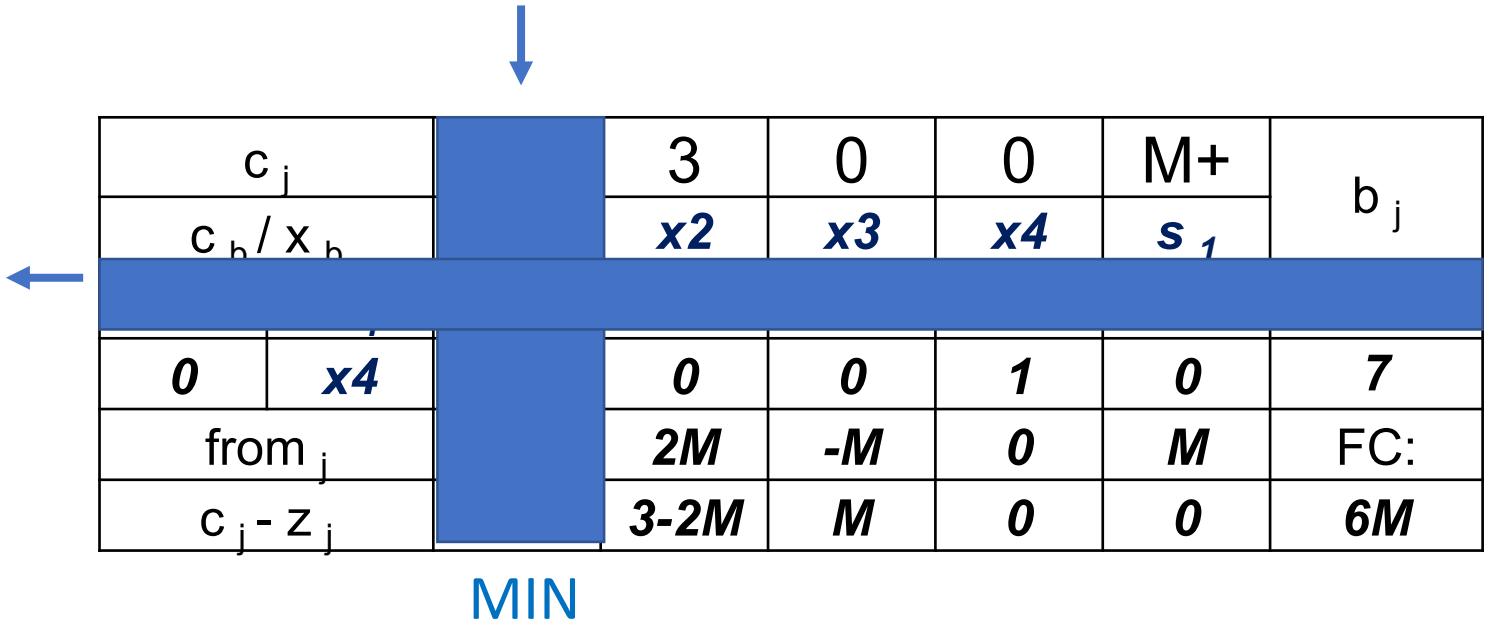
Simplex board model: I board

c_j		2	3	0	0	M+	b_j
c_b / x_b		***x1***	***x2***	***x3***	***x4***	***s_1***	
M	***s_1***	***3***	***2***	***-1***	***0***	***1***	***6***
0	***x4***	***1***	***0***	***0***	***1***	***0***	***7***
from $_j$		***3M***	***2M***	***-M***	***0***	***M***	FC:
$c_j - z_j$							***6M***

Optimality criterion: (for minimization) the solution is optimal
when **the values of optimality indicators are non-negative (0 and +)**

Task 3.2

Simplex board model: I board



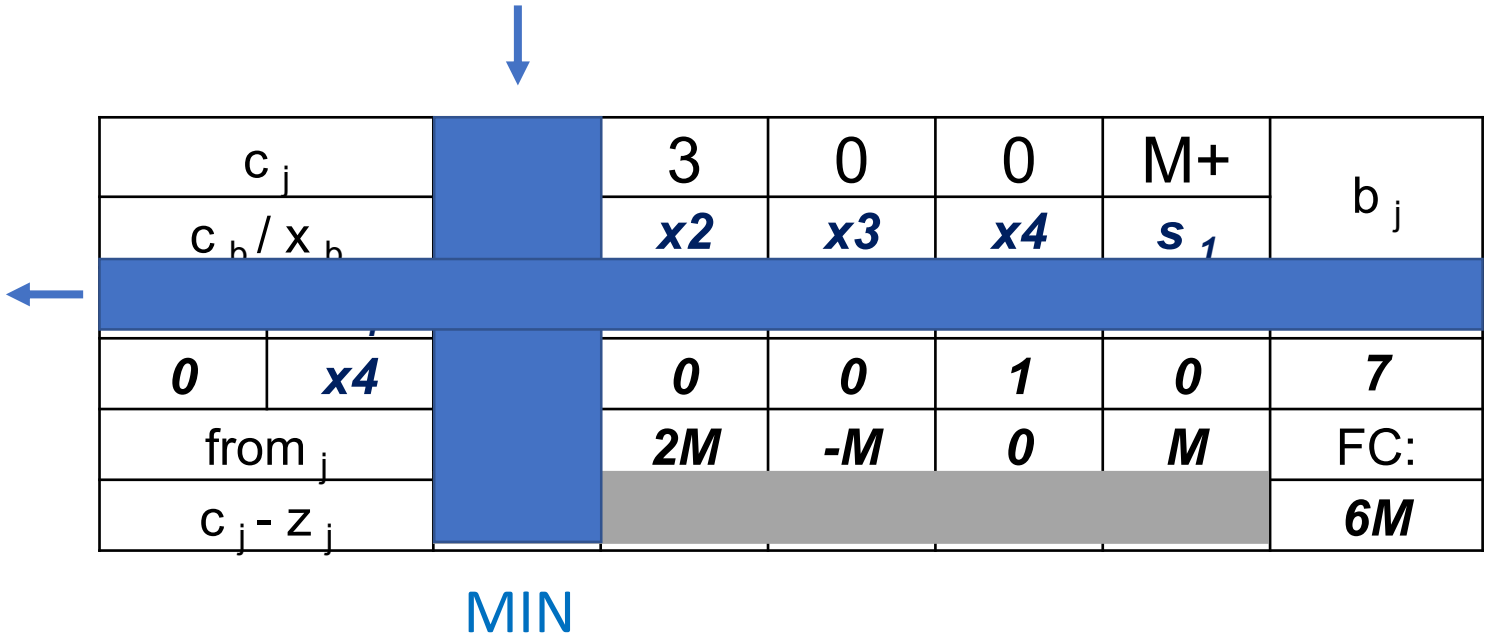
c_j			3	0	0	M+	b_j
c_b / x_b			*x2*	*x3*	*x4*	*s 1*	
0	*x4*		*0*	*0*	*1*	*0*	*7*
from $_j$			*2M*	*-M*	*0*	*M*	FC:
$c_j - z_j$			*3-2M*	*M*	*0*	*0*	*6M*

MIN

Min quotient enters the base:

Task 3.2

Simplex board model: I board



c_j			3	0	0	M+	b_j
c_b / x_b			*x2*	*x3*	*x4*	*s 1*	
0	*x4*		*0*	*0*	*1*	*0*	*7*
from $_j$			*2M*	*-M*	*0*	*M*	FC:
$c_j - z_j$							*6M*

MIN

Min quotient enters the base:

6/3= 2 min non-negative

7/1=7

Task 3.2

Simplex board model: I board

c_j			3	0	0	M+	b_j
c_b	x_b		*x2*	*x3*	*x4*	s_1	
0	*x4*		*0*	*0*	*1*	*0*	*7*
from $_j$			*2M*	*-M*	*0*	*M*	FC:
$c_j - z_j$			*3-2M*	*M*	*0*	*0*	*6M*

/(3)

(Step 1 - we determine the values for the first row of the new simplex table)

+

*(-1)

(Step 2 - we determine the values for the second and n-th row of the new simplex table)

Simplex board model: II board

c_j		2	3	0	0	M+	b_j
c_b	x_b	*x1*	*x2*	*x3*	*x4*	s_1	
2	*x1*	*1*	*2/3*	*-1/3*	*0*	*1/3*	*2*
0	*x4*	*0*	*-2/3*	*1/3*	*1*	*-1/3*	*5*
from $_j$							
$c_j - z_j$							

Task 3.2

Simplex board model: II board

c_j			3	0	0	M+	b_j
c_b	x_b		*x2*	*x3*	*x4*	s_1	
0	*x4*		*0*	*0*	*1*	*0*	*7*
from $_j$			*2M*	*-M*	*0*	*M*	FC:
$c_j - z_j$			*3-2M*	*M*	*0*	*0*	*6M*

c_j		2	3	0	0	M+	b_j
c_b	x_b	*x1*	*x2*	*x3*	*x4*	s_1	
2		*1*	*2/3*	*-1/3*	*0*	*1/3*	
0		*0*	*-2/3*	*1/3*	*1*	*-1/3*	
from $_j$		*2*	*4/3*	*-2/3*	*0*	*2/3*	FC:
$c_j - z_j$		*0*	*5/3*	*2/3*	*0*	*M-2/3*	

Optimal solution:

x1 = $_2$

x2 = $_0$

FC = 4

Task 3.3

3.3. Rozwiąż zagadnienie programowania liniowego metodą simpleks

$$FC: 24x_1 + 36x_2 + 12x_3 \rightarrow max$$

$$WO: \begin{cases} 2x_1 + 6x_2 + 4x_3 \leq 200 \\ 4x_1 + 2x_2 + 2x_3 \leq 100 \end{cases}$$

Task 3.3

Model

- FC: $_{24x1}$ + $_{36x2}$ □ max
- WO:
  - 2x1 $_{+}$ 6x2 + $_{4x3}$ ≤ $_{200}$
  - 4x1 $_{+}$ 2x2 + $_{2x3}$ ≤ $_{100}$
- WB: x_1 , $x_2 \geq 0$

Base Character

- FC: $24x_1 + 36x_2 12x_3 + \ldots$ □ max
- WO:
  - $2x1_{+} 6x2 + _{4x3} + \ldots = _{200}$
  - $4x_1 + 2x_2 + 2x_3 + \ldots = 100$
- WB: $x_1 : x_5 \geq 0$

Task 3.3

Model

- FC: $_{24x1} + _{36x2}$ □ max
- WO:
  - $2x1 _{+} 6x2 + _{4x3} \leq _{200}$
  - $4x1 _{+} 2x2 + _{2x3} \leq _{100}$
- WB: $x _{1}, x _{2} \geq 0$

Base Character

- FC: $24x _{1} + 36x _{2} + 12x _{3} 0x _{4} + 0x _{5}$ □ max
- WO:
  - $2x1 + _{6x2} + _{4x3} + _{x4} = 200$
  - $4x1 + _{2x2} + _{2x3} + _{x5} = 100$
- WB: $x _{1} : x _{5} \geq 0$

Task 3.3

Simplex board model: I board

c_j		24	36	12	0	0	b_j
c_b / x_b		***x1***	***x2***	***x3***	***x4***	***x5***	
0	***x4***	***2***	***6***	***4***	***1***	***0***	***200***
0	***x5***	***4***	***2***	***2***	***0***	***1***	***100***
from $_j$		***0***	***0***	***0***	***0***	***0***	FC:
$c_j - z_j$		***24***	***36***	***12***	***0***	***0***	***0***

- FC: $24x_1 + 36x_2 + 0x_4 + 0x_5$ → max
- WO:
  - $2x1 + {}_{6x2} + {}_{4x3} + {}_{x4} = 200$
  - $4x1 + {}_{2x2} + {}_{2x3} + {}_{x5} = 100$
- WB: $x_1 : x_5 \geq 0$

Task 3.3

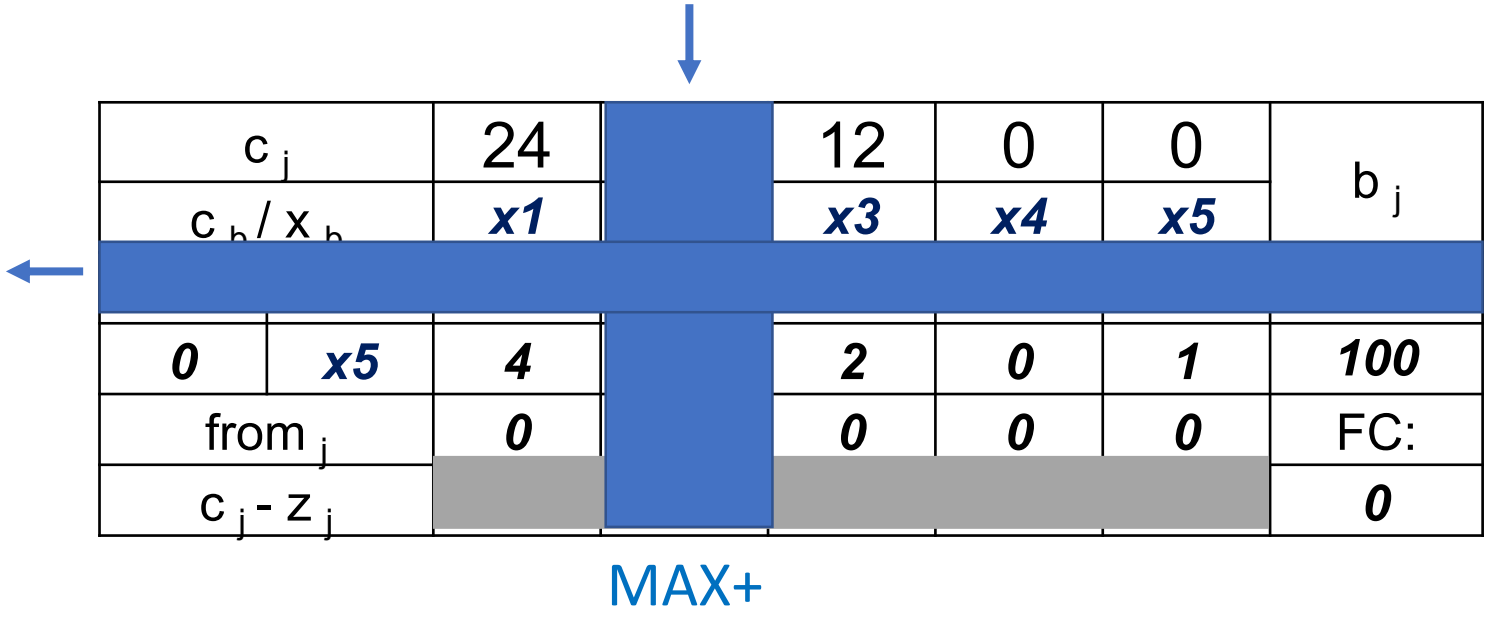
Simplex board model: I board

c_j		24	36	12	0	0	b_j
c_b / x_b		***x1***	***x2***	***x3***	***x4***	***x5***	
0	***x4***	***2***	***6***	***4***	***1***	***0***	***200***
0	***x5***	***4***	***2***	***2***	***0***	***1***	***100***
from $_j$		***0***	***0***	***0***	***0***	***0***	FC:
$c_j - z_j$							***0***

Optimality criterion: (for maximization) the solution is optimal
when **the values of optimality indicators are non-positive (0 and -)**

Task 3.3

Simplex board model: I board



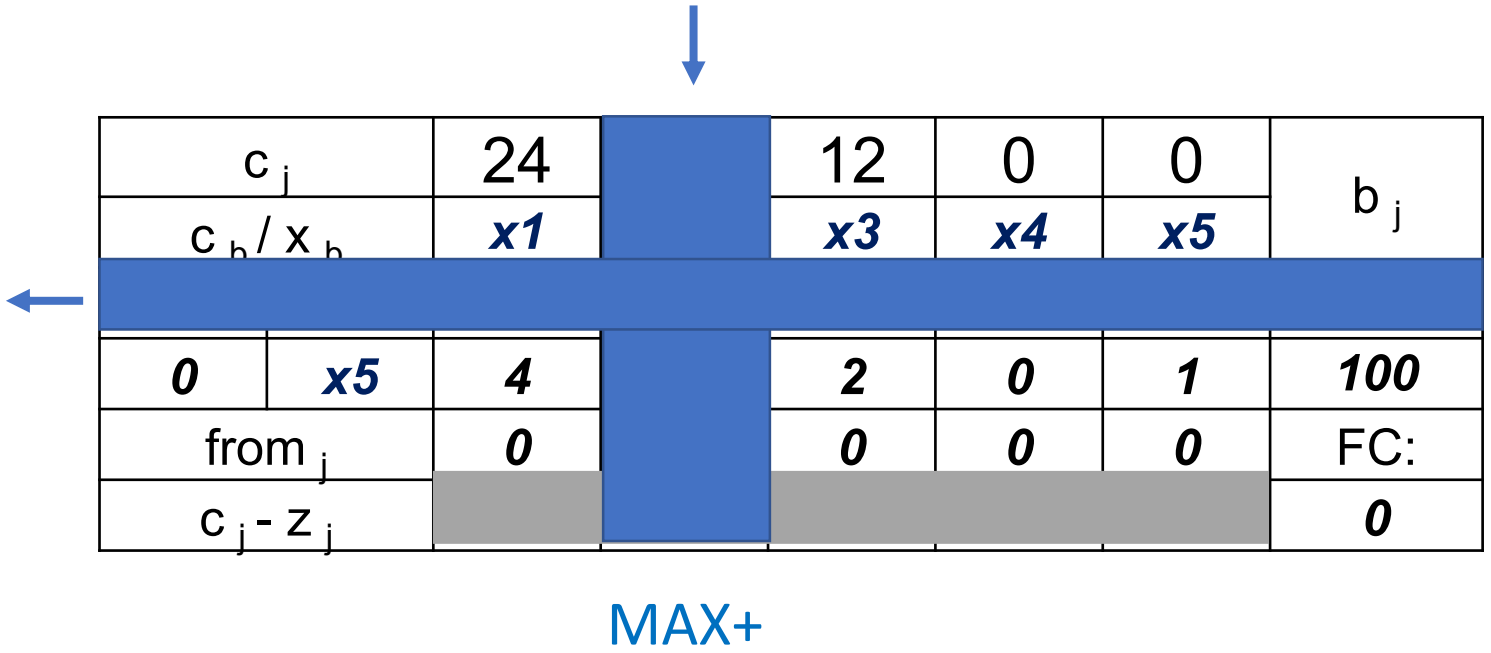
c_j		24		12	0	0	b_j
c_b / x_b		***x1***		***x3***	***x4***	***x5***	
0	***x5***	***4***		***2***	***0***	***1***	***100***
from $_j$		***0***		***0***	***0***	***0***	FC:
$c_j - z_j$							***0***

MAX+

Min quotient enters the base:

Task 3.3

Simplex board model: I board



c_j		24		12	0	0	b_j
c_b / x_b		***x1***		***x3***	***x4***	***x5***	
0	***x5***	***4***		***2***	***0***	***1***	***100***
from $_j$		***0***		***0***	***0***	***0***	FC:
$c_j - z_j$							***0***

MAX+

Min quotient enters the base:

200/6=33.3

100/2=50

Task 3.3

Simplex board model: II board

c_j		24		12	0	0	b_j
c_b / x_b		***x1***		***x3***	***x4***	***x5***	
0	***x5***	***4***		***2***	***0***	***1***	***100***
from $_j$		***0***		***0***	***0***	***0***	FC:
$c_j - z_j$		***24***		***12***	***0***	***0***	***0***

/(6) (1 step)

+

*(-2) (2 step)

c_j		24	36	12	0	0	b_j
c_b / x_b		***x1***	***x2***	***x3***	***x4***	***x5***	
36	***x2***	***1/3***	***1***	***2/3***	***1/6***	***0***	
0	***x5***		***0***			***1***	
from $_j$							
$c_j - z_j$							

Task 3.3

Simplex board model: II board

c_j		24		12	0	0	b_j
c_b / x_b		***x1***		***x3***	***x4***	***x5***	
0	***x5***	***4***		***2***	***0***	***1***	***100***
from $_j$		***0***		***0***	***0***	***0***	FC:
$c_j - z_j$		***24***		***12***	***0***	***0***	***0***

/(6) (1 step)

+

*(-2) (2 step)

c_j		24	36	12	0	0	b_j
c_b / x_b		***x1***	***x2***	***x3***	***x4***	***x5***	
36	***x2***	***1/3***	***1***	***2/3***	***1/6***	***0***	***100/3***
0	***x5***	***10/3***	***0***	***2/3***	***-2/6***	***1***	***100/3***
from $_j$		***12***	***36***	***24***	***6***	***0***	FC:
$c_j - z_j$							***1200***

Task 3.3

Simplex board model: II board

c_j			36	12	0	0	b_j
c_b / x_b			***x2***	***x3***	***x4***	***x5***	
36	***x2***		***1***	***2/3***	***1/6***	***0***	***100/3***
from $_j$			***36***	***24***	***6***	***0***	FC:
$c_j - z_j$			***0***	***-12***	***-6***	***0***	***1200***

c_j		24	36	12	0	0	b_j
c_b / x_b		***x1***	***x2***	***x3***	***x4***	***x5***	
	x						
	x						
from $_j$							
$c_j - z_j$							

Min quotient enters the base:

100/3/1/3=100

100/3/10/3=10

MAX+

Task 3.3

Simplex board model: II board

c_j			36	12	0	0	b_j
c_b / x_b			***x2***	***x3***	***x4***	***x5***	
36	***x2***		***1***	***2/3***	***1/6***	***0***	***100/3***
from $_j$			***36***	***24***	***6***	***0***	FC:
$c_j - z_j$			***0***	***-12***	***-6***	***0***	***1200***

/(10/3)
(1 step)

+

*(??) (2 step)

c_j		24	36	12	0	0	b_j
c_b / x_b		***x1***	***x2***	***x3***	***x4***	***x5***	
36	***x2***	***0***					
24	***x1***	***1***					
from $_j$							FC:
$c_j - z_j$							

Task 3.3

Simplex board model: II board

c_j			36	12	0	0	b_j
c_b / x_b			***x2***	***x3***	***x4***	***x5***	
36	***x2***		***1***	***2/3***	***1/6***	***0***	***100/3***
$from_j$			***36***	***24***	***6***	***0***	FC:
$c_j - z_j$			***0***	***-12***	***-6***	***0***	***1200***

/(10/3) (1 step)

+

*(-1/3) (2 step)

c_j		24	36	12	0	0	b_j
c_b / x_b		***x1***	***x2***	***x3***	***x4***	***x5***	
36	***x2***	***0***	***1***	***3/5***	***1/5***	***-1/10***	***30***
24	***x1***	***1***	***0***	***1/5***	***-1/10***	***3/10***	***10***
$from_j$		***24***	***36***	***132/5***	***24/5***	***18/5***	FC:
$c_j - z_j$							***1320***

Task 3.3

Simplex board model: III board

c_j		36	12	0	0	b_j
c_b / x_b		*x2*	*x3*	*x4*	*x5*	
36	*x2*	*1*	*2/3*	*1/6*	*0*	*100/3*
from $_j$		*36*	*24*	*6*	*0*	FC:
$c_j - z_j$		*0*	*-12*	*-6*	*0*	*1200*

c_j		24	36	12	0	0	b_j
c_b / x_b		*x1*	*x2*	*x3*	*x4*	*x5*	
36		*0*	*1*	*3/5*	*1/5*	*-1/10*	
24		*1*	*0*	*1/5*	*-1/10*	*3/10*	
from $_j$		*24*	*36*	*132/5*	*24/5*	*18/5*	FC:
$c_j - z_j$							

Optimal solution:

x1 = $_{10}$

x2 = $_{30}$

FC = 1320

Homework (Group 2)

3.4. Solve the linear programming problem using the simplex method

$$FC: 12x_1 + 18x_2 \rightarrow min$$

$$WO: \begin{cases} 6x_1 + 3x_2 \geq 120 \\ x_1 + 3x_2 \geq 60 \end{cases}$$

Homework (Group 1)

3.5. Solve the linear programming problem using the simplex method

$$FC: -x_1 + 3x_2 - 2x_3 \to max$$

$$WO: \begin{cases} 3x_1 - x_2 + 2x_3 \leq 7 \\ -2x_1 + 4x_2 \leq 12 \\ -4x_1 + 3x_2 + 8x_3 \leq 10 \end{cases}$$

Additional task (Group 2)

Solve the following linear programming problem using the simplex algorithm:

Objective function: $FC = x_1 + 2x_2 + x_3 \rightarrow \min$

Subject to: $\begin{cases} x_1 + x_2 - x_3 = 46 \\ x_1 + 3x_2 + 3x_3 \geq 100 \\ -2x_2 + x_3 \leq 36 \\ x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0 \end{cases}$

Additional task (Group 1)

Solve the following linear programming problem using the simplex algorithm:

Objective function: $FC = 2x_1 - 3x_2 + x_3 \rightarrow \max$

Subject to: $\begin{cases} x_1 - 2x_2 + 2x_3 \leq 5 \\ x_2 + 3x_3 \geq 3 \\ x_1 + x_2 - 2x_3 = 20 \\ x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0 \end{cases}$

Sources:

- Materials from the subject posted on the eNauczanie website : Z. Kędra
- Z. Jędrzejczyk, J. Skrzypek, K. Kukuła, A. Walkosz : Operational research in examples and tasks. PWN. Warsaw, 1996
- M. Glinka: Elements of operational research in transport. Radom University of Technology Publishing House. Radom, 2009
- Other books and textbooks on Operational Research available in the PG Library