ADTRANS
ADVANCED DATA ANALYSIS FOR TRANSPORTATION AND MOBILITY ACADEMIC TEACHERS

Operational Research - Transport Algorithm

Lukasz Jelinski

Co-funded by the European Union

Task 5.1

1.Four bakeries located in the city are supplied with flour from two warehouses situated on the outskirts. The flour resources in the warehouses are: warehouse A – 130 tons, warehouse B – 200 tons, and the demand of the bakeries is respectively: 80, 120, 70, and 60 tons. The cost of delivering flour to the bakeries depends only on the distances given in the table below (in km). Determine a transportation plan that ensures minimization of flour delivery costs.

Warehouses	Bakeries 1	Bakeries 2	Bakeries 3	Bakeries 4
A	25	24	28	13
B	17	30	15	26

Task 1

Transport volume table

(1 step – number of base variables)

We calculate the number of base nodes (how many cells will transports occur in)

m + n – 1 2 + 4 – 1 = 5

m – number of suppliers = 2
n – number of recipients = 4

We define supply and demand constraints **(Step 2 – is the issue balanced?)**

Demand = 80+120+70+60 = 330
Supply = 130 +200 = 330

Demand =
Supply ⍰

the issue is balanced

Task 1

Transport cost matrix

25	*24*	*28*	*13*
17	*30*	*15*	*26*

Transport volume table

(3 step)

(initial feasible solution)

- Northwest angle method
- The minimum element method of the cost matrix

	1	2	3	4	
A N					130
D B					200
	80	120	70	60	

1) Min(130.80) etc.
2) I mark the nodes * where the transports are

Task 1

Transport volume table

(4 step - iterations)

80*	50*		
	70*	70*	60*

Table of optimality indicators (iteration 1)

25 0*	24 0*	28	13
17	30 0*	15 0*	26 0*

1) Building a new optimality indicator board

- we introduce u_i, v_j and define the pointers

optimality for nodes

$e_{ij} = u_i + v_j + c_{ij} = 0$ - assumption for nodes

$u_i + v_j$ - we calculate

C_{ij} - from the cost table

we get m+n-1 equations and m+n unknowns

- infinitely many solutions

hence I assume that one variable e.g. $u_1 = 0$

$e_{11} = u_1 + v_1 + 25 = 0$

$e_{12} = u_1 + v_2 + 24 = 0$

$e_{22} = u_2 + v_2 + 30 = 0$

$e_{23} = u_2 + v_3 + 15 = 0$

Task 1

Transport volume table

(4 step - iterations)

*80**	*50**		
	*70**	*70**	*60**

Table of optimality indicators (iteration 1)

25 *0**	24 *0**	28	13
17	30 *0**	15 *0**	26 *0**

1) Building a new optimality indicator board

- we introduce u_i, v_j and define the pointers optimality for nodes

  $e_{ij} = u_i + v_j + c_{ij} = 0$ ⍰ assumption for nodes

  $u_i + v_j$ ⍰ we calculate

  C_{ij} ⍰ from the cost table

  we get m+n-1 equations and m+n unknowns
  - infinitely many solutions
  hence I assume that one variable e.g. $u_1 = 0$

$e11 = u1 + v1 + 25 = 0$

$e12 = u1 + v2 + 24 = 0$

$e22 = u2 + v2 + 30 = 0$

$e23 = u2 + v3 + 15 = 0$

$u1 = 0$

$v1 = -25$

$v2 = -24$

$u2 = -6$

$v3 = -9$

$v4 = -20$

Task 1

Transport volume table

*80**	*50**		
	*70**	*70**	*60**

Table of optimality indicators (iteration 1)

25 *0**	*24* *0**	*28* *19*	*13* *-7*
17 *-14*	*30* *0**	*15* *0**	*26* *0**

1) Building a new optimality indicator board
- we introduce u_i, v_j and define the pointers optimality for nodes
- we define other indicators

$e_{13} = u_1 + v_3 + 28 = 0-9+28=19$

e14 = u1 + v4 + 13 = -7

e21 = u2 + v1 + 17 = -14

u1 = 0

v1 = -25

v2 = -24

u2 = -6

v3 = -9

v4 = -20

Task 1

Transport volume table

*80**	*50**		
	*70**	*70**	*60**

1) Building a new optimality indicator board

- we introduce u_i, v_j and define the pointers optimality for nodes
- we define other indicators

- We check if the solution is optimal

Table of optimality indicators (iteration 1)

25 *0**	24 *0**	28 *19*	13 *-7*
17 *-14*	30 *0**	15 *0**	26 *0**

The solution is not optimal
- there are negative values

$e_{13} = u_1 + v_3 + 28 = 0-9+28=19$

$e14 = u1 + v4 + 13 = -7$

$e21 = u2 + v1 + 17 = -14$

u1 = 0
v1 = -25
v2 = -24
u2 = -6
v3 = -9
v4 = -20

Task 1

Transport volume table

80* (-)	50* (+)		
(+)	70* (-)	70*	60*

Table of optimality indicators (iteration 1)

25 0*	24 0*	28 19	13 -7
17 -14 *	30 0*	15 0*	26 0*

I establish criteria for entering the database

- I find the element <0 with the smallest value
- I enter the corresponding node into the database

I set the criterion for exiting the database (a node removed from the database – building a so-called "cycle")

- I go back to the transport quantity table and start the cycle (+) from there where a new node was created.
- the cycle is based on nodes
- we assign alternate signs (-) (+) to subsequent vertices in the cycle

Task 1

Transport volume table

Min(-) = 70

80* (-)	50* (+)		
(+)	70* (-)	70*	60*

Table of optimality indicators (iteration 1)

25 0*	24 0*	28 19	13 -7
17 -14 *	30 0*	15 0*	26 0*

- I remove the node (-) which has the smallest value
- I subtract the value of the deleted node from the remaining (-)
- to the remaining (+) I add the value of the deleted node

In this way, a new node is created for the future table of transport volumes

Task 1

Table of transport quantities (iter . 2)

*10**	*120**		
*70**		*70**	*60**

Table of optimality indices (iter . 2)

0 *0**	*0* *0**	*19*	*-7*
-14 *0 **	*0*	*0* *0**	*0* *0**

1) **Building a new optimality indicator board**

- **we introduce u_i, v_j and define the pointers optimality for nodes**

$e_{ij} = u_i + v_j + c_{ij} = 0$? assumption
$u_i + v_j$? we calculate
C_{ij} ? for the incoming node
= value of the previous indicator (i.e. -14)
remaining = 0
we get m+n-1 equations and m+n unknowns
- infinitely many solutions
hence I assume that one variable e.g. u_1 =0

$e11 = u1 + v1 + 0 = 0$
$e12 = u1 + v2 + 0 = 0$
$e21 = u2 + v1 - 14 = 0$
$e23 = u2 + v3 + 0 = 0$
$e24 = u2 + v4 + 0 = 0$

u1 = 0
u2 = 14
v1 = 0
v2 = 0
v3 = -14
v4 = -14

Task 1

Table of transport quantities (iter . 2)

*10**	*120**		
*70**		*70**	*60**

1) Building a new optimality indicator board
- we define other indicators

Table of optimality indices (iter . 2)

0 *0**	*0* *0**	*19* *5*	*-7* *-21*
-14 *0* *	*0* *14*	*0* *0**	*0* *0**

e13 = u1 + v3 + 19 = 5
e14 = u1 + v4 -7 = -21
e22 = u2 + v2 + 0 = 14

u1 = 0
u2 = 14
v1 = 0
v2 = 0
v3 = -14
v4 = -14

Task 1

Table of transport quantities (iter . 2)

10*	120*		
70*		70*	60*

Table of optimality indices (iter . 2)

The solution is not optimal - there are negative values

0 0*	0 0*	19 5	-7 -21
-14 0 *	0 14	0 0*	0 0*

e13 = u1 + v3 + 19 = 5
e14 = u1 + v4 -7 = -21
e22 = u2 + v2 + 0 = 14

u1 = 0
u2 = 14
v1 = 0
v2 = 0
v3 = -14
v4 = -14

Task 1

Table of transport quantities (iter . 2)

Min(-) = 10

10* -	120*		+
70* +		70*	60* -

Table of optimality indices (iter . 2)

0*	0*	5	-21
0*	14	0*	0*

I establish criteria for entering the database

- I find the element <0 with the smallest value
- I enter the corresponding node into the database

I set the criterion for exiting the database (removed from the database - cycle construction)

- I go back to the transport quantity table and start the cycle (+) from there where a new node was created.
- the cycle is based on nodes
- we assign alternate signs (-) (+) to subsequent vertices in the cycle

- I remove the node (-) which has the smallest value
- I subtract the value of the deleted node from the remaining (-)
- to the remaining (+) I add the value of the deleted node

Task 1

Table of transport quantities (iter . 3)

0	120*		10*
80*	0	70*	50*

Table of optimality indices (iter . 3)

0	0 *	5	-21 *
0 *	14	0 *	0 *

Task 1

Table of transport quantities (iter . 3)

0	*120**		*10**
*80**	*0*	*70**	*50**

Table of optimality indices (iter . 3)

0	*0* *0**	5	*-21* *0* *
0 *0**	*14*	*0* *0**	*0* *0**

1) Building a new optimality indicator board
- we introduce u_i, v_j and define the pointers optimality for nodes
  $e_{ij} = u_i + v_j + c_{ij} = 0$ ⍰ assumption
  $u_i + v_j$ ⍰ we calculate
  C_{ij} ⍰ for the incoming node
  = value of the previous indicator (i.e. -21)
  remaining = 0
  we get m+n-1 equations and m+n unknowns
  - infinitely many solutions
  hence I assume that one variable e.g. $u_1 = 0$

$e12 = u1 + v2 + 0 = 0$

$e14 = u1 + v4 - 21 = 0$

$e21 = u2 + v1 + 0 = 0$

$e23 = u2 + v3 + 0 = 0$

$e24 = u2 + v4 + 0 = 0$

Task 1

Table of transport quantities (iter . 3)

0	*120**		*10**
*80**	*0*	*70**	*50**

Table of optimality indices (iter . 3)

0	*0* *0**	5	*-21* *0* *
0 *0**	*14*	*0* *0**	*0* *0**

1) **Building a new optimality indicator board**

- **we introduce u_i , v_j and define the pointers optimality for nodes**

  $e_{ij} = u_i + v_j + c_{ij} = 0$ ⍰ assumption
  $u_i + v_j$ ⍰ we calculate
  C_{ij} ⍰ for the incoming node
  = value of the previous indicator (i.e. -21)
  remaining = 0
  we get m+n-1 equations and m+n unknowns
  - infinitely many solutions
  hence I assume that one variable e.g. $u_1 = 0$

e12 = u1 + v2 + 0 = 0
e14 = u1 + v4 - 21 = 0
e21 = u2 + v1 + 0 = 0
e23 = u2 + v3 + 0 = 0
e24 = u2 + v4 + 0 = 0

u1 = 0
u2 = -21
v1 = 21
v2 = 0
v3 = 21
v 4 = 21

Task 1

Table of transport quantities (iter . 3)

0	*120**		*10**
*80**	*0*	*70**	*50**

1) **Building a new optimality indicator board**

- **we define other indicators**

Table of optimality indices (iter . 3)

0 *21*	0 *0**	5 *26*	-21 *0**
0 *0**	14 *-7*	0 *0**	0 *0**

e11 = u1 + v1 + 0 = 21
e13 = u1 + v3 + 5 = 26
e22 = u2 + v2 + 14 = -7

u1 = 0
u2 = -21
v1 = 21
v2 = 0
v3 = 21
v 4 = 21

Task 1

Table of transport quantities (iter . 3)

0	120*		10*
80*	0	70*	50*

Table of optimality indices (iter . 3)

0 21	0 0*	5 26	-21 0*
0 0*	14 -7	0 0*	0 0*

The solution is not optimal
- there are negative values

Task 1

Table of transport quantities (iter . 3)

Min(-) = 50

0	*120** (-)		*10** (+)
*80**	*0* (+)	*70**	*50** (-)

Table of optimality indices (iter . 3)

0 / *21*	0 / *0**	5 / *26*	-21 / *0**
0 / *0**	14 / *-7*	0 / *0**	0 / *0**

I establish criteria for entering the database

- I find the element <0 with the smallest value
- I enter the corresponding node into the database

I set the criterion for exiting the database (removed from the database - cycle construction)

- I go back to the transport quantity table and start the cycle (+) from there where a new node was created.
- the cycle is based on nodes
- we assign alternate signs (-) (+) to subsequent vertices in the cycle

- I remove the node (-) which has the smallest value
- I subtract the value of the deleted node from the remaining (-)
- to the remaining (+) I add the value of the deleted node

Task 1

Table of transport quantities (iter . 4)

0	70*		60*
80*	50*	70*	0

Table of optimality indices (iter . 4)

21	0 *	26	0 *
0 *	-7 *	0 *	0

Task 1

Table of transport quantities (iter . 4)

0	*70**		*60**
*80**	*50**	*70**	*0*

Table of optimality indices (iter . 4)

21	*0* *0**	*26*	*0* *0**
0 *0**	*-7* *0* *	*0* *0**	*0*

1) **Building a new optimality indicator board**

- **we introduce u_i, v_j and define the pointers optimality for nodes**

$e_{ij} = u_i + v_j + c_{ij} = 0$? assumption
$u_i + v_j$? we calculate
C_{ij} ? for the incoming node
= value of the previous indicator (i.e. -7)
remaining = 0
we get m+n-1 equations and m+n unknowns
- infinitely many solutions
hence I assume that one variable e.g. $u_1 = 0$

e12 = u1 + v2 + 0 = 0
e14 = u1 + v4 + 0 = 0
e21 = u2 + v1 + 0 = 0
e22 = u2 + v2 -7 = 0

u1 = 0
u2 = 7
v1 = -7
v2 = 0
v3 = -7
v4 = 0

Task 1

Table of transport quantities (iter . 4)

0	70*		60*
80*	50*	70*	0

Table of optimality indices (iter . 4)

21 14	0 0*	26 19	0 0*
0 0*	-7 0*	0 0*	0 7

The solution is optimal
- all non-negative values

Task 1

Table of transport quantities (iter . 4)

0	*70**		*60**
*80**	*50**	*70**	*0*

Freight Cost Table

25	*24*	*28*	*13*
17	*30*	*15*	*26*

In order to minimize flour delivery costs, you should provide:

- **From warehouse A: 70 t to bakery 2 and 60 t to bakery 4**
- **From warehouse B: 80 t to bakery 1, 50 t to bakery 2, 70 t to bakery 3**

We calculate the cost of deliveries (transports)

FC = $c_{12} * x_{12} + c_{14} * x_{14} + \ldots + c_{23} * x_{23}$

=24*70+13*60+17*80+30*50+15*70=6370

The minimum cost of transport will be 6370 km

Task 2

2.Three mines: K_1 ,K_2 ,K_3 supply coal to five depots, located in different places. Each depot can receive 400 tons of coal per month, while the extraction capacities of the mines are: K_1 600 – tons, K_2 and K_3 700 – tons each per month. The unit costs of transportation (in PLN per ton, depending on distance) are shown in the table below. Determine a transportation plan that ensures the minimization of coal transport costs.

Mines	Depot D1	Depot D2	Depot D3	Depot D4	Depot D5
M1	14	5	9	24	15
M2	30	24	11	8	19
m3	9	22	15	7	18

Task 3

3.For the transportation problem described in the table below, construct a mathematical model and then determine a feasible solution using the northwest corner method and the minimum element method. Compare the objective function value for both these solutions.

Suppliers	**Receivers**				**a_i**
	1	**2**	**3**	**4**	
S1	18	15	25	15	500
S2	21	24	10	19	400
S3	14	30	19	17	300
b_j	300	200	300	200	

Task 4

4.Four production plants A, B, C, D supply four receivers O1, O2, O3, O4. The production capacities of the plants are: 10, 40, 30, and 30 respectively. The demand of the receivers is: 20, 20, 30, and 20 respectively. The unit transportation costs between the plants and the receivers are shown in the table. Determine the optimal transportation plan that minimizes transport costs.

Producers	**O1**	**O2**	**O3**	**O4**
A	4	3	1	3
B	3	5	3	1
C	3	6	2	1
D	1	3	4	5

Task 4

Transport volume table **(1 step – number of base variables)**

We calculate the number of base nodes (how many cells will transports occur in)

m + n – 1 4 + 4 – 1 = 7

m – number of suppliers = 4
n – number of recipients = 4

We define supply and demand constraints **(Step 2 – is the issue balanced?)**

Demand = 10+40+30+30 = 110
Supply = 20 + 20 + 30 + 20 = 90

Demand > Supply ⍰ **the issue is unbalanced**

Task 4

Transport volume table

(we add recipient O5 – fictitious about demand 20 and costs transport =0)

	O1	O2	O3	O4	O5	
A N D B						10
						40
C						30
D						30
	20	20	30	20	20	

Transport cost matrix

4	*3*	*1*	*3*	*?*
3	*5*	*3*	*1*	*?*
3	*6*	*2*	*1*	*?*
1	*3*	*4*	*5*	*?*

1) e.g. Northwest angle method
2) I mark the nodes * where the transports are

Task 4

Transport volume table

(we add recipient O5 – fictitious about demand 20 and costs transport =0)

	O1	O2	O3	O4	O5	
A	*10*	*0*	*0*	*0*	*0*	10
B	*10*	*20*	*10*	*0*	*0*	40
C	*0*	*0*	*20*	*10*	*0*	30
D	*0*	*0*	*0*	*10*	*20*	30
	20	20	30	20	20	

Transport cost matrix

4	*3*	*1*	*3*	*0*
3	*5*	*3*	*1*	*0*
3	*6*	*2*	*1*	*0*
1	*3*	*4*	*5*	*0*

1) e.g. Northwest angle method
2) I mark the nodes * where the transports are
3) I check if the solution is not degenerate

Sources:

- Materials from the subject posted on the eNauczanie website : Z. Kędra
- Z. Jędrzejczyk, J. Skrzypek, K. Kukuła, A. Walkosz : Operational research in examples and tasks. PWN. Warsaw, 1996
- M. Glinka: Elements of operational research in transport. Radom University of Technology Publishing House. Radom, 2009
- Other books and textbooks on Operational Research available in the PG Library