AdTrans project , 2023-1-PL01-KA220-HED-000158917

ADTRANS
ADVANCED DATA ANALYSIS FOR TRANSPORTATION AND MOBILITY ACADEMIC TEACHERS

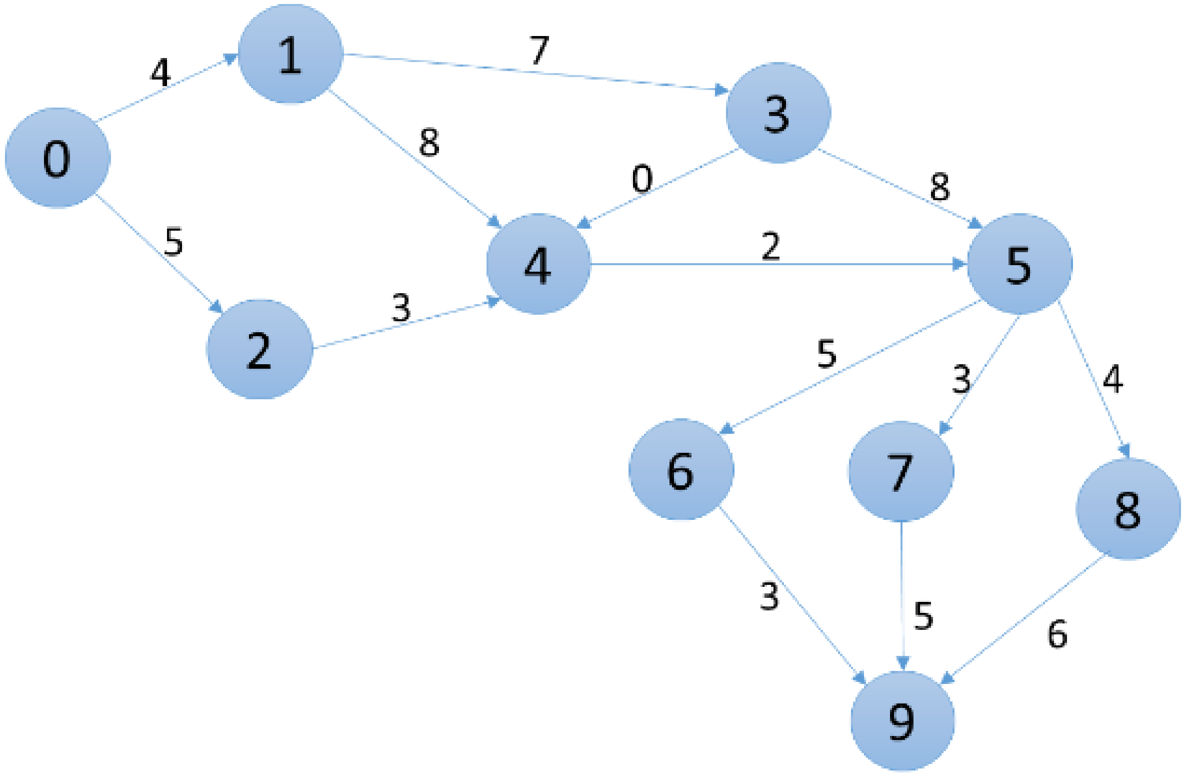
Operational Research - Dijkstra's Algorithm

Lukasz Jelinski

Co-funded by the European Union

Task 1

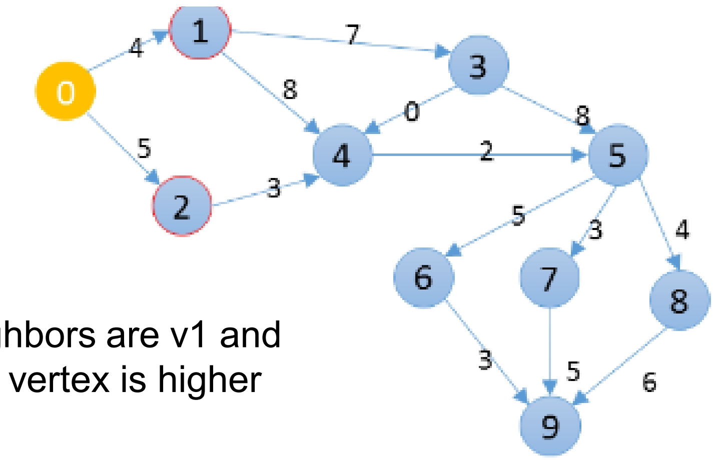
Example: Find the shortest path between vertices 0 and 9 using Dijkstra's algorithm



Task 1

We create sets: S (elements processed by the algorithm) and Q (unprocessed elements). Initially, set S is empty, and all elements are in set Q. We build a table in which, in each row, we place: the cost $d(u)$ of moving between the starting vertex and a given vertex u, and in the second row, the number of the predecessor vertex $p(u)$ .The first table: enter the cost in the starting vertex as 0, and for the other vertices, for now, $+\infty$.

u	0	1	2	3	4	5	6	7	8	9
d(u)	0	∞	∞	∞	∞	∞	∞	∞	∞	∞
p(u)	-	-	-	-	-	-	-	-	-	-



The operations for vertex 0 entering S:
Vertex 0 enters set S because its cost is the lowest. Its neighbors are v1 and v2. We check whether the current cost of moving to a given vertex is higher than the cost for the currently analyzed path to that vertex:

$$d[1] > d[0] + 4? \text{ YES } => d[1] = d[0] + 4 = 0 + 4 = 4$$

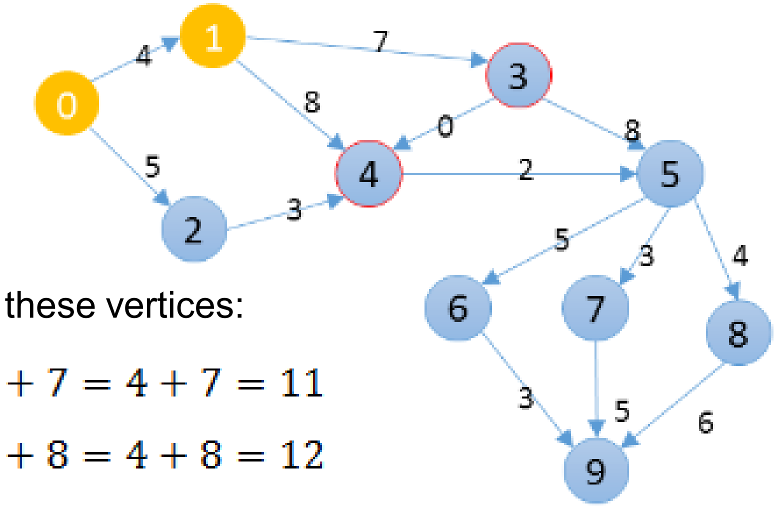
$$d[2] > d[0] + 5? \text{ YES } => d[2] = d[0] + 5 = 0 + 5 = 5$$

The calculated costs are lower, so we enter them into the table. The lowest cost occurs for vertex v1, so we introduce it to set S.

Task 1

The calculated costs are lower, so we enter them into the table. The lowest cost occurs for vertex v1, so we introduce it to set S.

u	0	1	2	3	4	5	6	7	8	9
d(u)	0	4	5	∞	∞	∞	∞	∞	∞	∞
p(u)	-	0	0	-	-	-	-	-	-	-



The neighbors of vertex v1 are v3 and v4 – we check the cost for these vertices:

$$d[3] > d[1] + 7?\ \text{YES} \quad => \quad d[3] = d[1] + 7 = 4 + 7 = 11$$

$$d[4] > d[1] + 8?\ \text{YES} \quad => \quad d[4] = d[1] + 8 = 4 + 8 = 12$$

We enter the new costs and predecessors into the table:

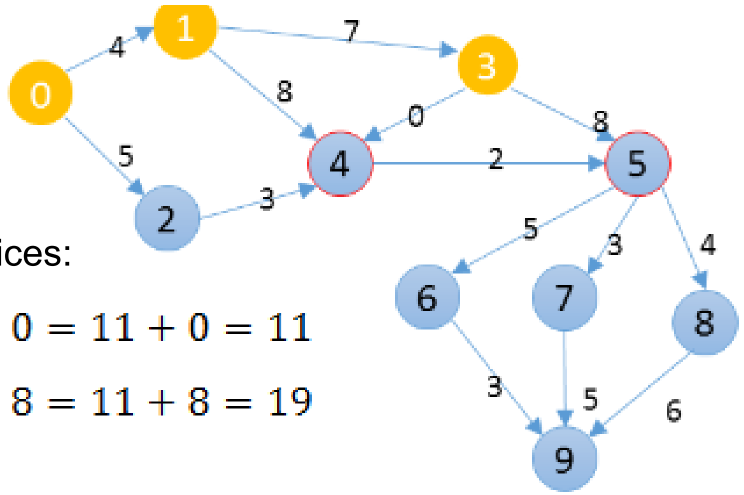
u	0	1	2	3	4	5	6	7	8	9
d(u)	0	4	5	11	12	∞	∞	∞	∞	∞
p(u)	-	0	0	1	1	-	-	-	-	-

Task 1

u	0	1	2	3	4	5	6	7	8	9
d(u)	0	4	5	11	12	∞	∞	∞	∞	∞
p(u)	-	0	0	1	1	-	-	-	-	-

Vertex w3 can be added to set S, because its cost is the lowest possible (there is no shorter path to w3).

u	0	1	2	3	4	5	6	7	8	9
d(u)	0	4	5	11	12	∞	∞	∞	∞	∞
p(u)	-	0	0	1	1	-	-	-	-	-



The neighbors of w3 are w4 and w5 – check the cost for these vertices:

$d[4] > d[3] + 0?$ YES $=> d[4] = d[3] + 0 = 11 + 0 = 11$

$d[5] > d[3] + 8?$ YES $=> d[5] = d[3] + 8 = 11 + 8 = 19$

We enter the new costs into the table:

u	0	1	2	3	4	5	6	7	8	9
d(u)	0	4	5	11	11	19	∞	∞	∞	∞
p(u)	-	0	0	1	3	3	-	-	-	-

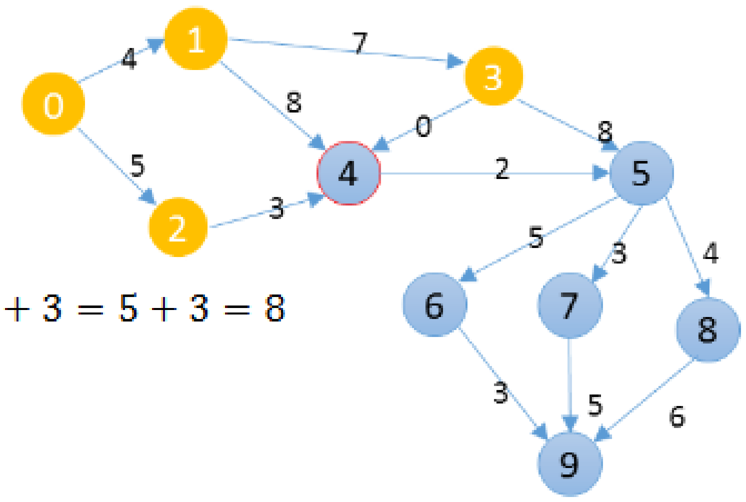
Task 1

We enter the new costs into the table.

u	0	1	2	3	4	5	6	7	8	9
d(u)	0	4	5	11	11	19	∞	∞	∞	∞
p(u)	-	0	0	1	3	3	-	-	-	-

The next lowest cost is for vertex w2 – we move it to set S and check the time to reach its neighbor. The only neighbor of w2 is w4.

u	0	1	2	3	4	5	6	7	8	9
d(u)	0	4	5	11	11	19	∞	∞	∞	∞
p(u)	-	0	0	1	3	3	-	-	-	-



Check the neighbor:

$d[4] > d[2] + 3$ YES $=> d[4] = d[2] + 3 = 5 + 3 = 8$

New table:

u	0	1	2	3	4	5	6	7	8	9
d(u)	0	4	5	11	8	19	∞	∞	∞	∞
p(u)	-	0	0	1	2	3	-	-	-	-

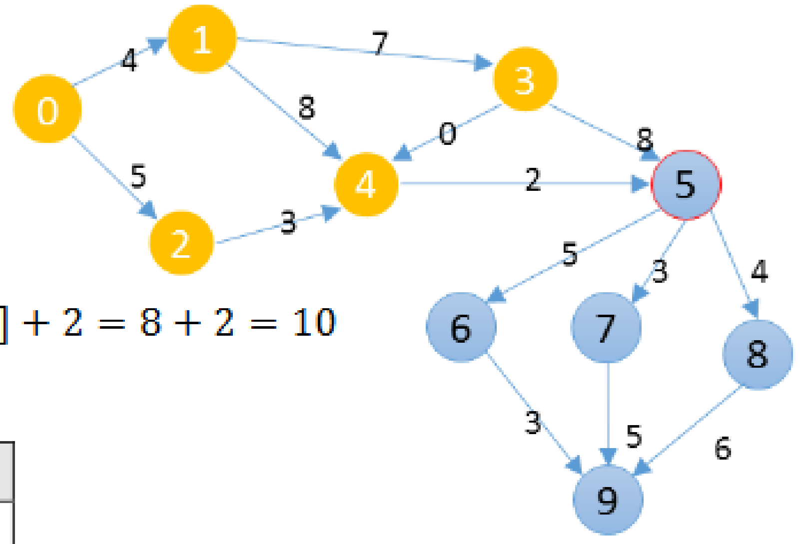
Task 1

New table:

u	0	1	2	3	4	5	6	7	8	9
d(u)	0	4	5	11	8	19	∞	∞	∞	∞
p(u)	-	0	0	1	2	3	-	-	-	-

We move w4 to set S (its cost is as low as possible – we've checked all possible options). Its only neighbor is w5:

u	0	1	2	3	4	5	6	7	8	9
d(u)	0	4	5	11	8	19	∞	∞	∞	∞
p(u)	-	0	0	1	2	3	-	-	-	-



We check the condition for w5:

$$d[5] > d[4] + 2\text{: YES} \quad => \quad d[5] = d[4] + 2 = 8 + 2 = 10$$

New table:

u	0	1	2	3	4	5	6	7	8	9	
d(u)	0	4	5	11	8	10	∞	∞	∞	∞	
p(u)	-	0	0	1	2	4	-	-	-	-	

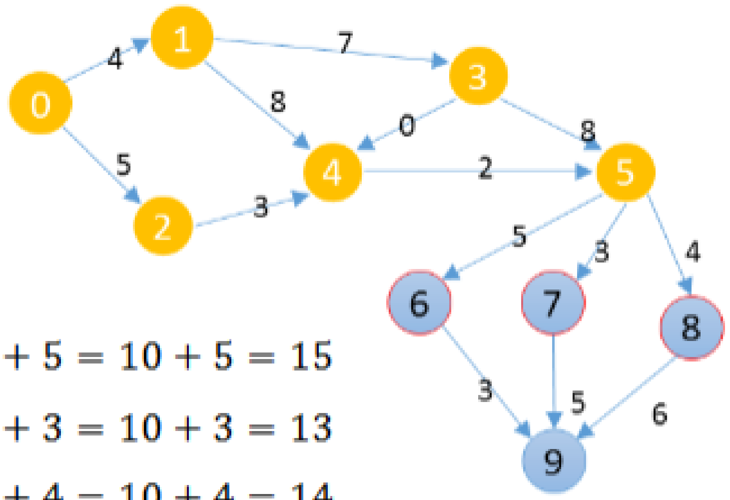
Task 1

New table:

u	0	1	2	3	4	5	6	7	8	9	
d(u)	0	4	5	11	8	10	∞	∞	∞	∞	

We may add w5 to set S, because we've checked all possible options to reach it. w5 has three neighbors: w6, w7, and w8.

u	0	1	2	3	4	5	6	7	8	9
d(u)	0	4	5	11	8	10	∞	∞	∞	∞
p(u)	-	0	0	1	2	4	-	-	-	-



Check the respective conditions:

$d[6] > d[5] + 5$? YES => $d[6] = d[5] + 5 = 10 + 5 = 15$

$d[7] > d[5] + 3$? YES => $d[7] = d[5] + 3 = 10 + 3 = 13$

$d[8] > d[5] + 4$? YES => $d[8] = d[5] + 4 = 10 + 4 = 14$

New table:

u	0	1	2	3	4	5	6	7	8	9
d(u)	0	4	5	11	8	10	15	13	14	∞
p(u)	-	0	0	1	2	4	5	5	5	-

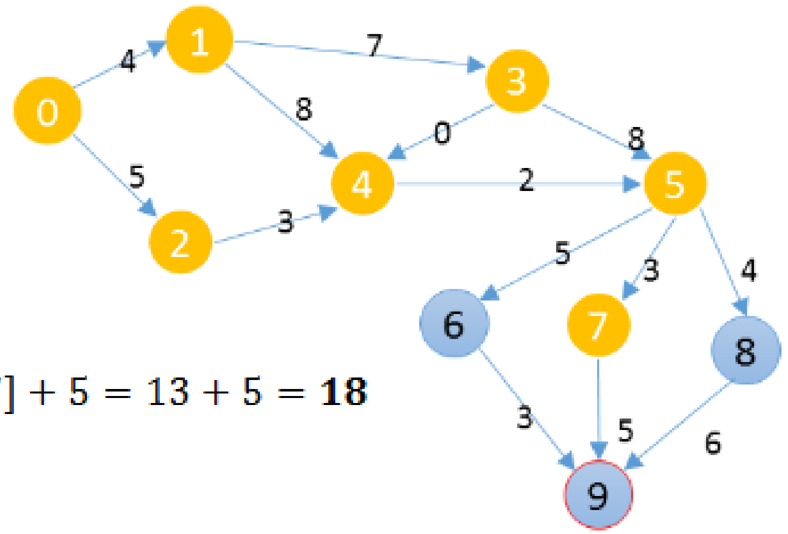
Task 1

New table:

u	0	1	2	3	4	5	6	7	8	9
d(u)	0	4	5	11	8	10	15	13	14	∞
p(u)	-	0	0	1	2	4	5	5	5	-

The lowest cost occurs for w7; we move it to set S.

u	0	1	2	3	4	5	6	7	8	9
d(u)	0	4	5	11	8	10	15	13	14	∞
p(u)	-	0	0	1	2	4	5	5	5	-



We check its neighbor: w9.

$$d[9] > d[7] + 5? \text{ YES } \Rightarrow d[9] = d[7] + 5 = 13 + 5 = \mathbf{18}$$

New table:

u	0	1	2	3	4	5	6	7	8	9
d(u)	0	4	5	11	8	10	15	13	14	18
p(u)	-	0	0	1	2	4	5	5	5	7

Task 1

New table:

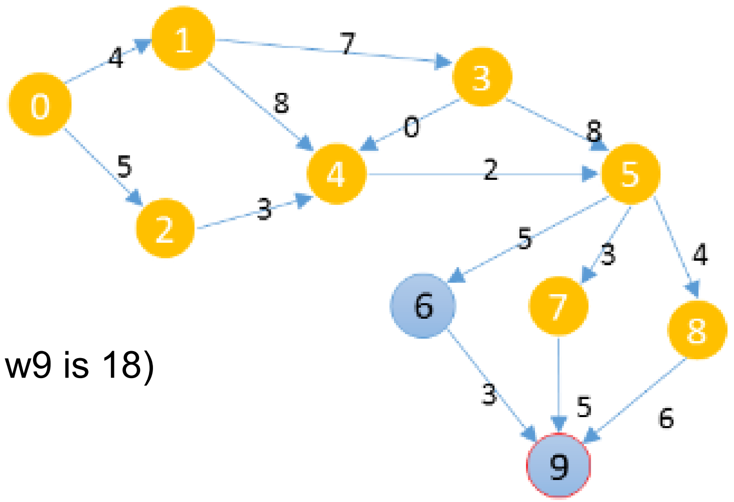
u	0	1	2	3	4	5	6	7	8	9
d(u)	0	4	5	11	8	10	15	13	14	18
p(u)	-	0	0	1	2	4	5	5	5	7

Next, the lowest cost is at w8. We move it to set S and check the condition:

$d[9] > d[8] + 6?$ NO => leave it (the value for w9 is 22)

The table now looks as follows:

u	0	1	2	3	4	5	6	7	8	9
d(u)	0	4	5	11	8	10	15	13	14	18
p(u)	-	0	0	1	2	4	5	5	5	7



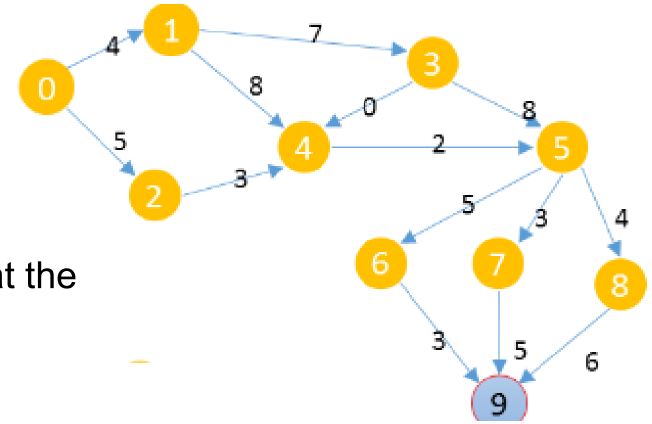
Similarly, check the condition for w6:

$d[9] > d[6] + 3?$ NO => leave it (the value for w9 is 18)

Task 1

w6 and w9 are moved to set S. Thus, we have processed all vertices with the algorithm and set Q is empty.

u	0	1	2	3	4	5	6	7	8	9
d(u)	0	4	5	11	8	10	15	13	14	18
p(u)	-	0	0	1	2	4	5	5	5	7



To find the solution, we trace back from the end, looking at the predecessors in the table.

u	0	1	2	3	4	5	6	7	8	9
d(u)	0	4	5	11	8	10	15	13	14	18
p(u)	-	0	0	1	2	4	5	5	5	7

or

u	0	1	2	3	4	5	6	7	8	9
d(u)	0	4	5	11	8	10	15	13	14	18
p(u)	-	0	0	1	2	4	5	5	5	6

Task 1

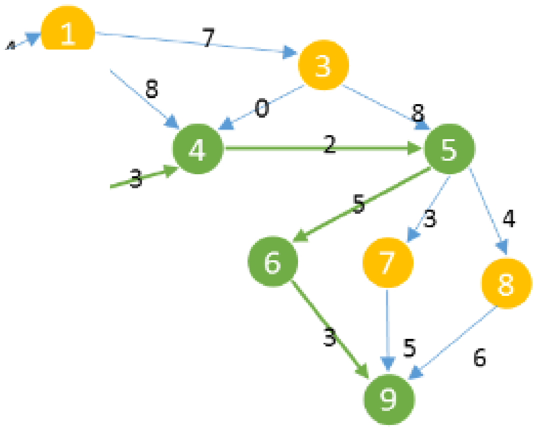
We search for the solution from the end – looking at the predecessors in the table.

u	0	1	2	3	4	5	6	7	8	9
d(u)	0	4	5	11	8	10	15	13	14	18
p(u)	-	0	0	1	2	4	5	5	5	7

or

u	0	1	2	3	4	5	6	7	8	9
d(u)	0	4	5	11	8	10	15	13	14	18
p(u)	-	0	0	1	2	4	5	5	5	6

To summarize, the shortest path occurs along the vertices:
0-2-4-5-7-9
or
0-2-4-5-6-9
The cost for both paths is 18.



Task 2

2.Find the shortest path connecting events 1 and 7 in a directed graph defined by the table, documenting the subsequent steps of Dijkstra's algorithm. Draw the graph and the path found.

Activity	Cost
1-2	3
1-3	6
2-4	2
2-5	5
3-6	7
4-5	3
4-6	1
5-7	6
6-7	4

Sources:

- Materials from the subject posted on the eNauczanie website : Z. Kędra
- Z. Jędrzejczyk, J. Skrzypek, K. Kukuła, A. Walkosz : Operational research in examples and tasks. PWN. Warsaw, 1996
- M. Glinka: Elements of operational research in transport. Radom University of Technology Publishing House. Radom, 2009
- Other books and textbooks on Operational Research available in the PG Library