

Operational Research - Graphs

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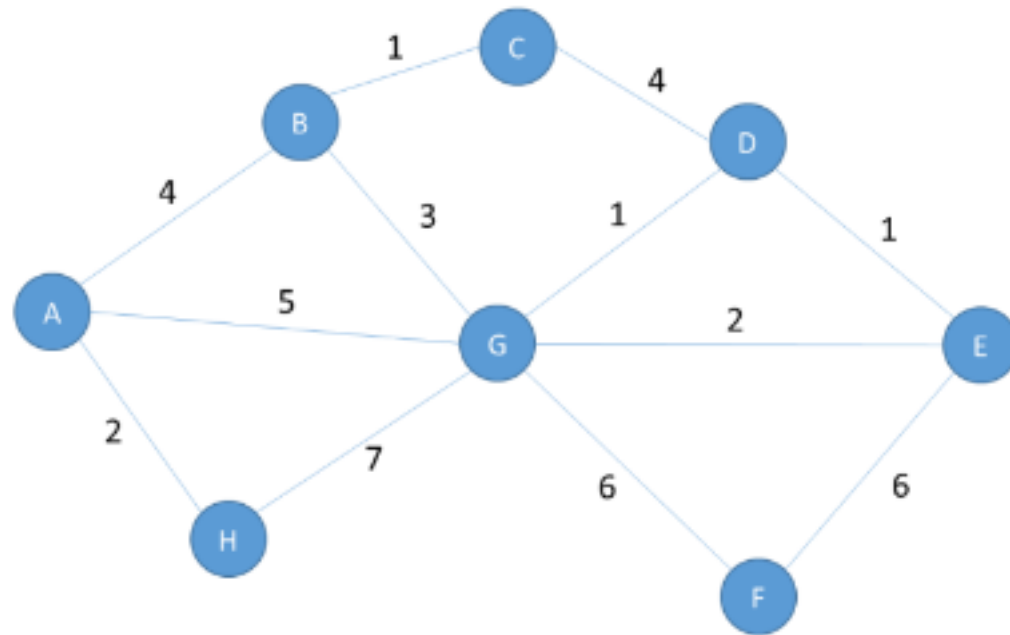


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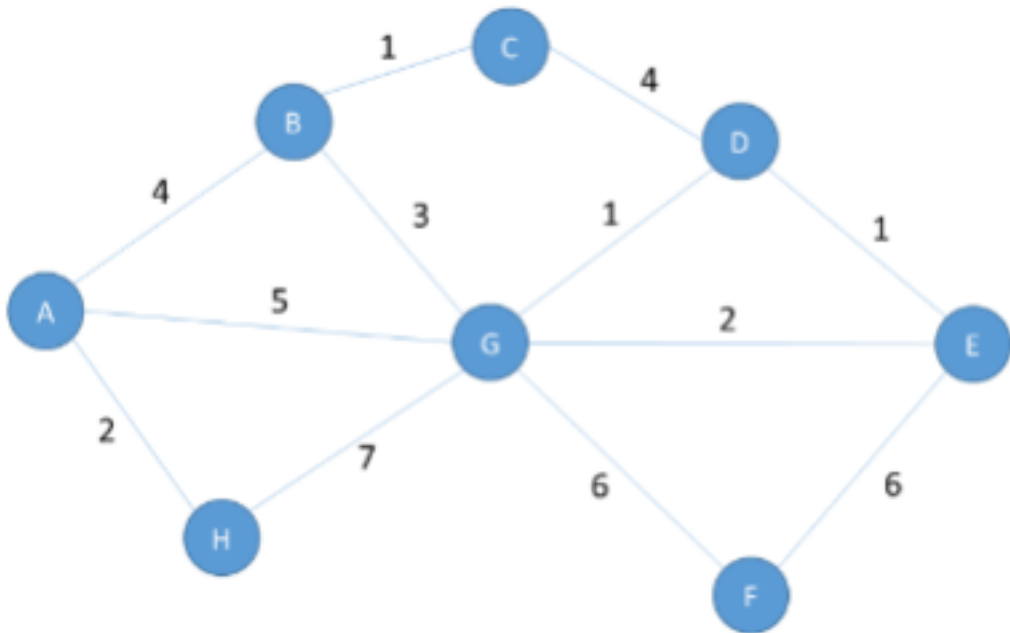
Task 3a

3 a) Solve the Chinese Postman Problem for the graph:





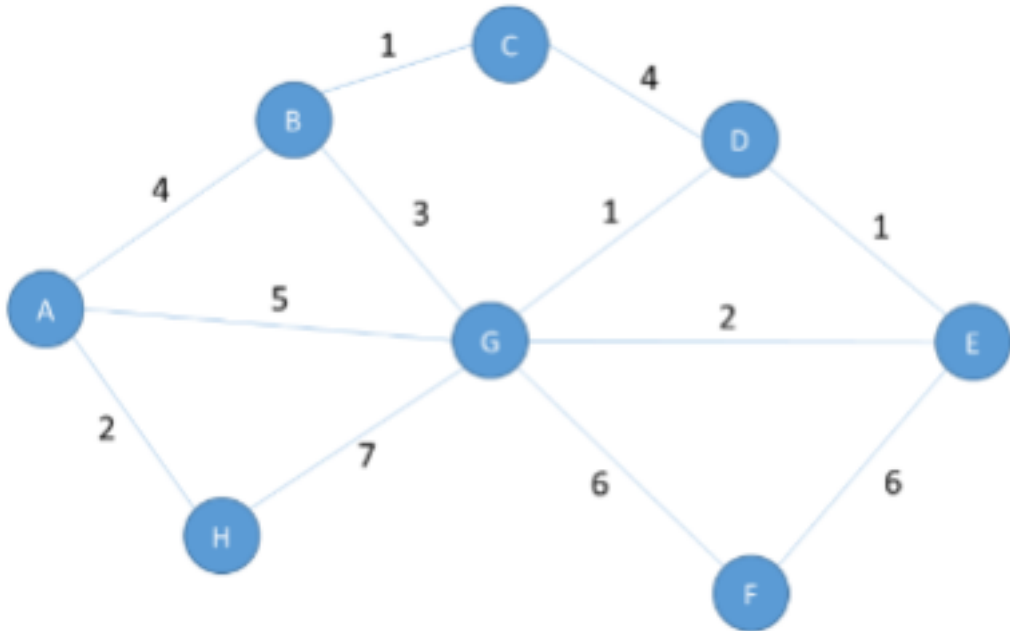
Task 3



1. Is the graph Eulerian? – No, some vertices have odd degree
2. We are looking for vertices with odd degree: ABDE
3. We connect them in pairs and look for the shortest paths



Task 3



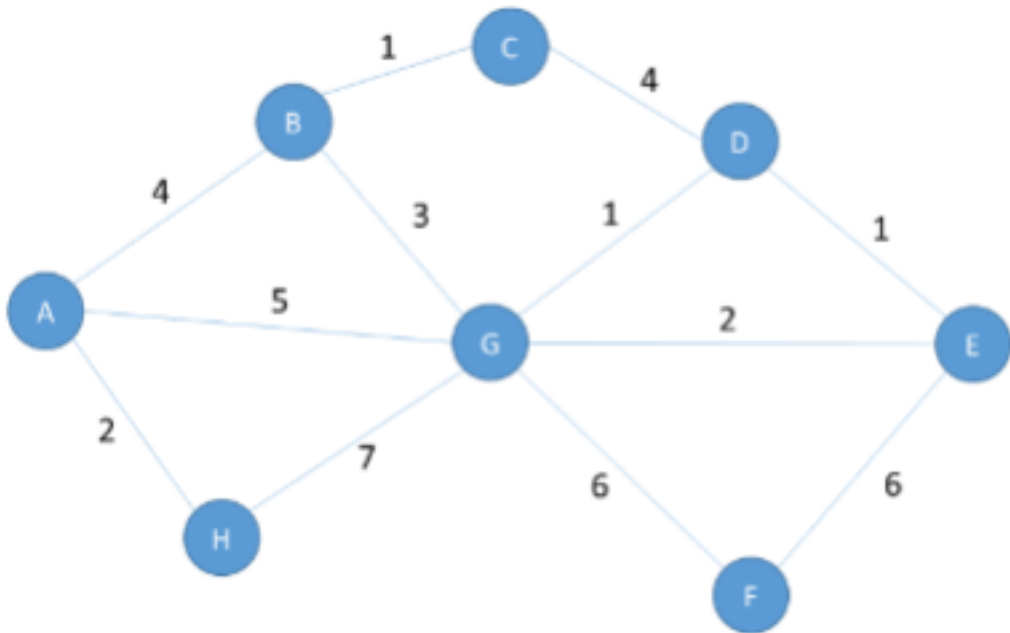
Lowest cost???

Couples	The lowest cost
AB DE	4 1
AD* BE*	6 5
AE* BD*	7 4

* indirect connection



Task 3



4. We duplicate the paths on AB and DE – now the graph is Eulerian

5. Total cost in the problem = sum of edges + sum of duplicate edges

Sum of edges = 42
Total Edge Area = 5



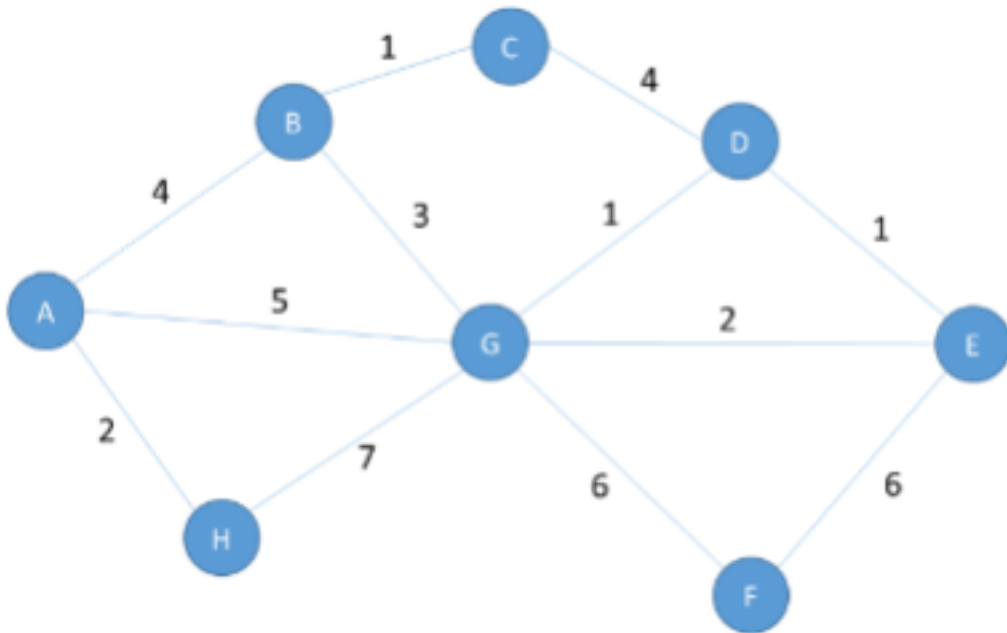
Total cost = 42 + 5 = 47

This is the minimum cost of transition from the starting point through all edges and return to the starting point

6. Route planning



Task 3



4. We duplicate the paths on AB and DE – now the graph is Eulerian

5. Total cost in the problem = sum of edges + sum of duplicate edges

Sum of edges = 42
Total Edge Area = 5



Total cost = 42+5 47

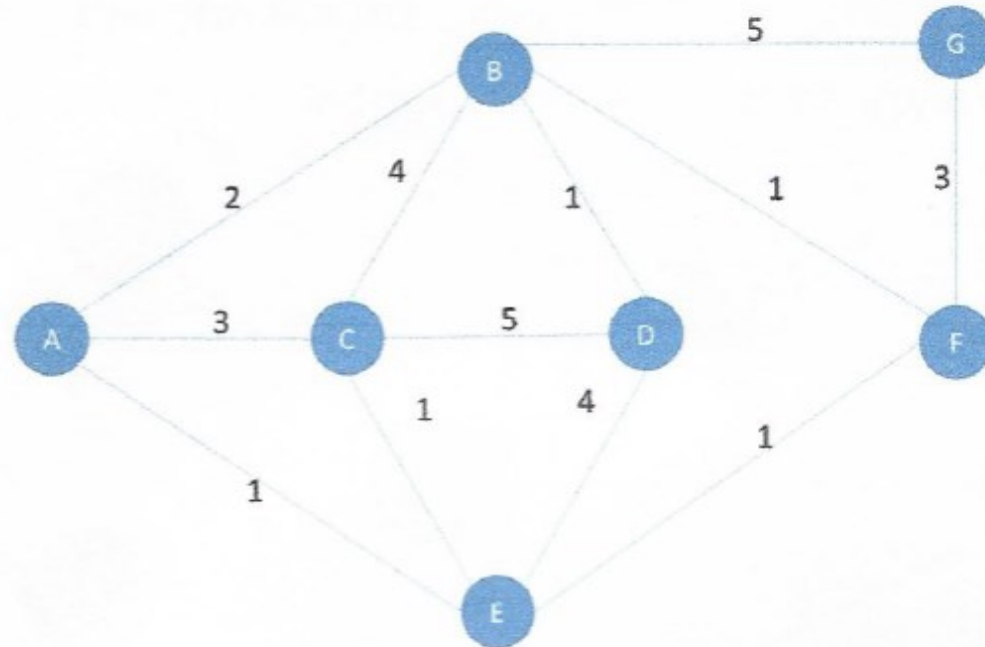
This is the minimum cost of transition from the starting point through all edges and return to the starting point

6. Route planning
AHGFEGABGDEDC



Task 3b

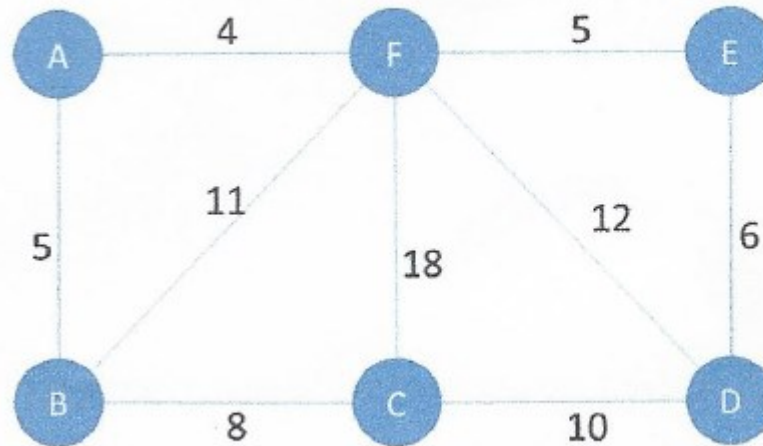
3 b) Solve the Chinese Postman Problem for the graph:





Task 3c

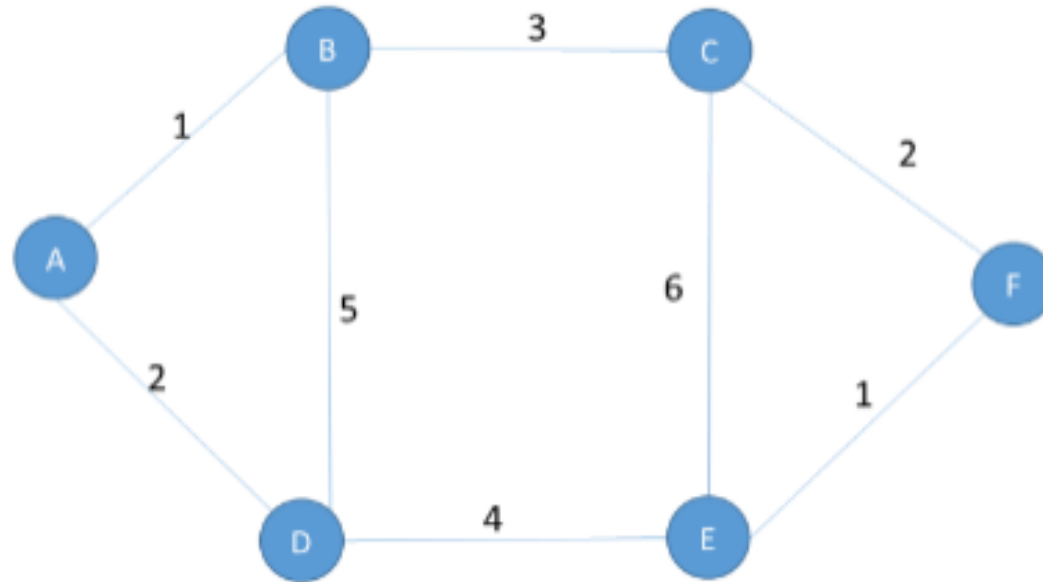
3 c) Solve the Chinese Postman Problem for the graph:





Task 3d

3 d) Solve the Chinese Postman Problem for the graph:



Task 4


Determine the maximum number of trucks that we can send from point s to point t , taking into account road capacity.

Fulkerson algorithm

Assumptions:

- The flow in the channel cannot exceed its capacity.
- Residual network - difference: capacity - flow

we have  some spare capacity

 when the stock is 0, we can't let any more flow through it

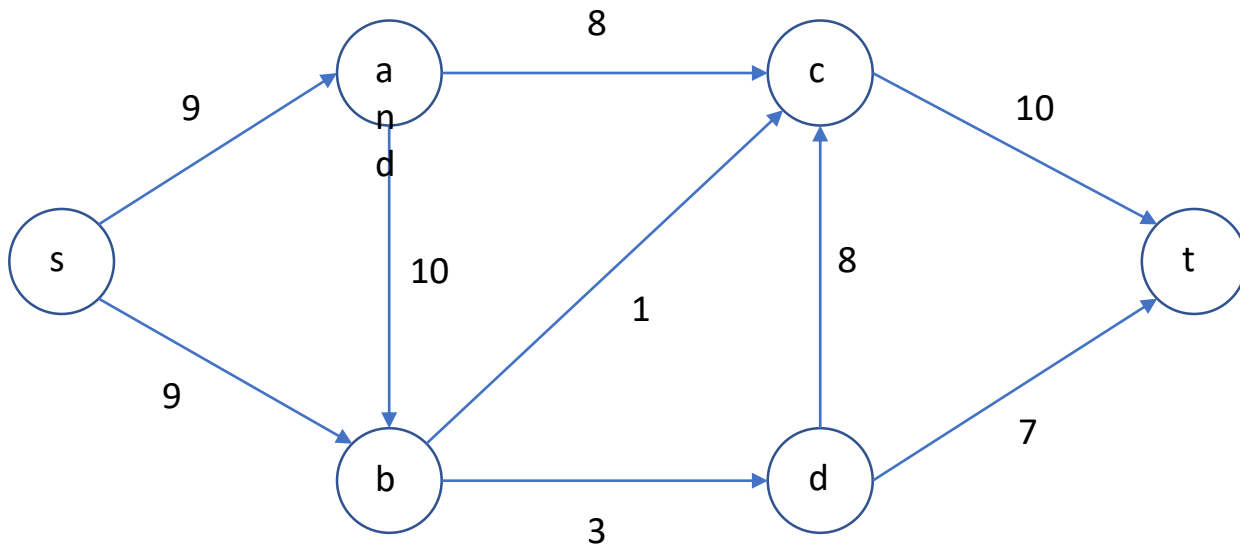
Network flow: the sum of flows from source s to all other nodes of the network

maximum flow = minimum throughput



Task 4

Determine the maximum number of trucks that we can send from point s to point t, taking into account road capacity.

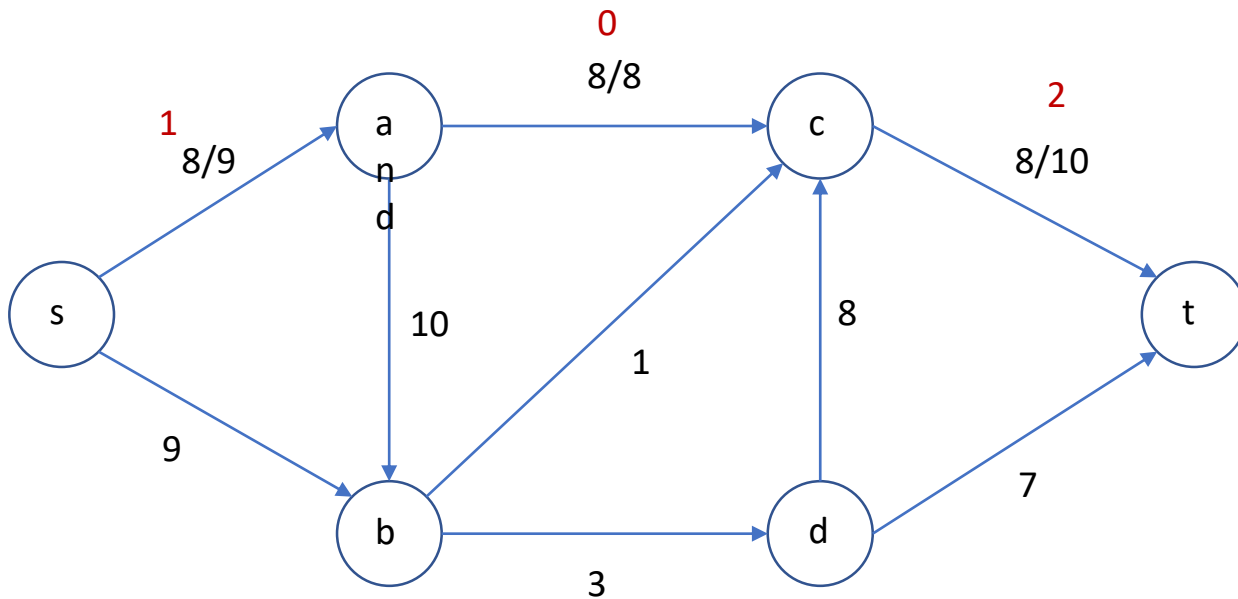


- 1) We are looking for a path with the smallest number of edges
e.g. $s - a - c - t$
 - I determine the smallest throughput = $8 \Rightarrow f(u,v)=8$
 - We add it from the maximum flow $F_m = 0+8$ and update the graph



Task 4

Determine the maximum number of trucks that we can send from point *s* to point *t*, taking into account road capacity.

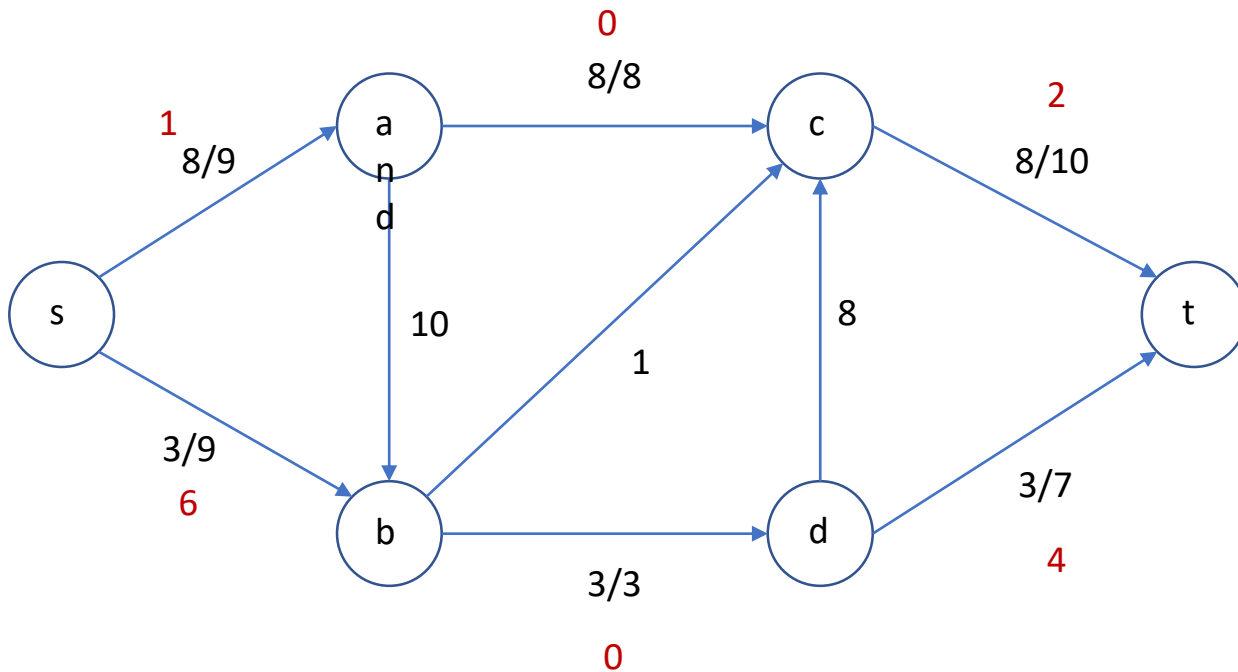


- 2) We look for the next path with the smallest number of edges e.g. *s* – *b* – *d* – *t*
- I determine the smallest throughput = 3 \square $f(u,v)=3$
 - We add it from the maximum flow $F_m = 0+8+3$ and update the graph



Task 4

Determine the maximum number of trucks that we can send from point s to point t , taking into account road capacity.

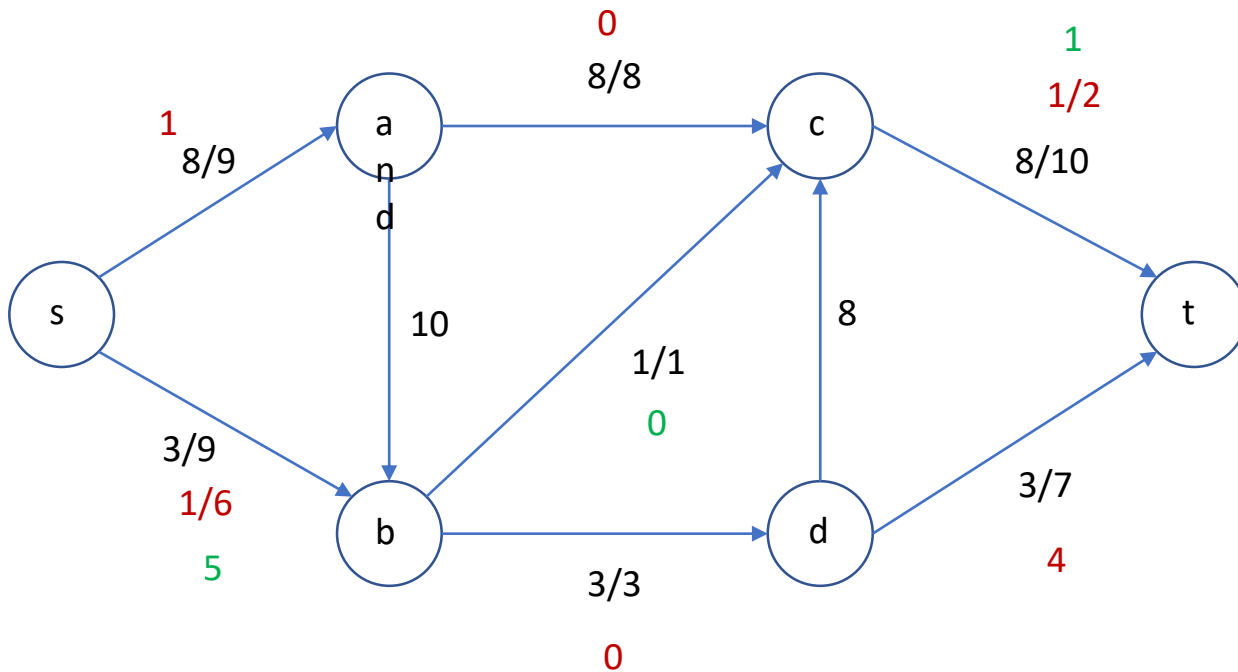


- 3) We look for the next path with the smallest number of edges e.g. $s - b - c - t$
- I determine the smallest throughput = 1 $\Rightarrow f(u,v)=1$
 - We add it from the maximum flow $F_m = 0+8+3+1$ and update the graph



Task 4

Determine the maximum number of trucks that we can send from point s to point t , taking into account road capacity.

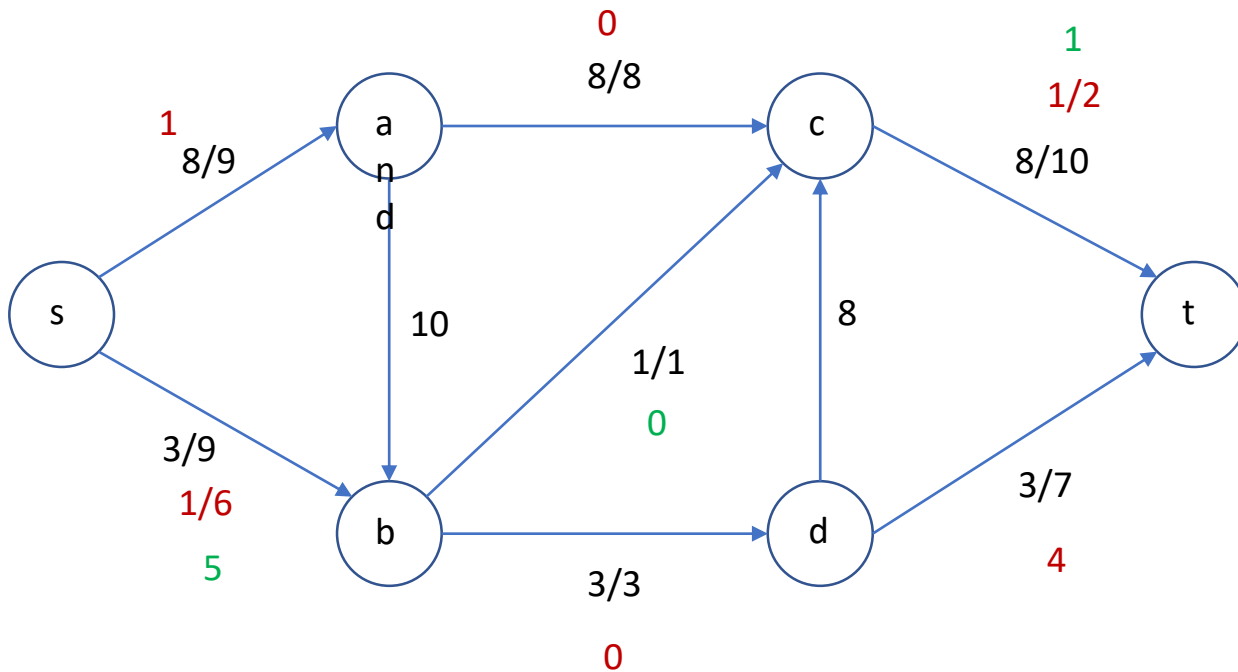


- 4) We look for the next path with the smallest number of edges e.g. $s - b - c - t$
- I determine the smallest throughput = 1 $\Rightarrow f(u,v)=1$
 - We add it from the maximum flow $F_m = 0+8+3+1$ and update the graph



Task 4

Determine the maximum number of trucks that we can send from point s to point t, taking into account road capacity.

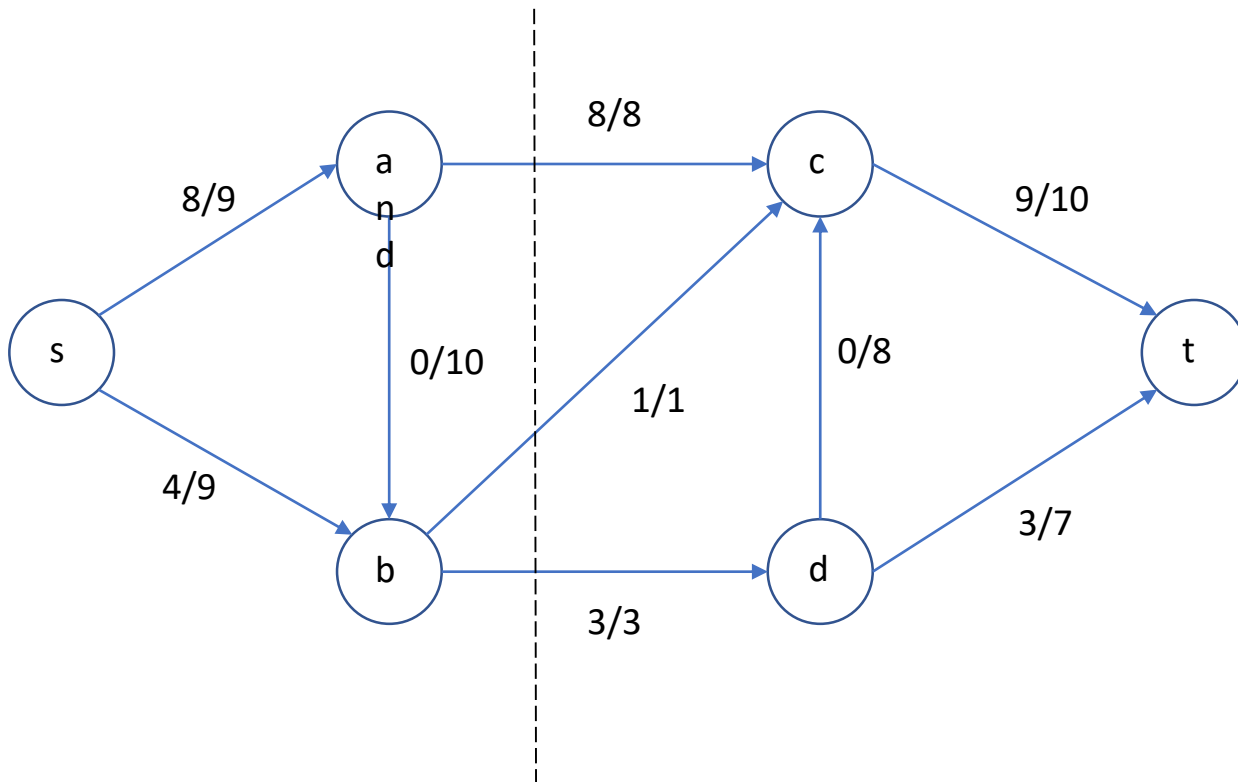


- 5) No paths - end of algorithm
- 6) Maximum flow is
 $F_m = 0+8+3+1=12$
- 7) Flow distribution: GRAF
- 8) Minimum network cross-section?



Task 4

Determine the maximum number of trucks that we can send from point s to point t, taking into account road capacity.



5) No paths - end of algorithm

6) Maximum flow is

$$F_m = 0 + 8 + 3 + 1 = 12$$

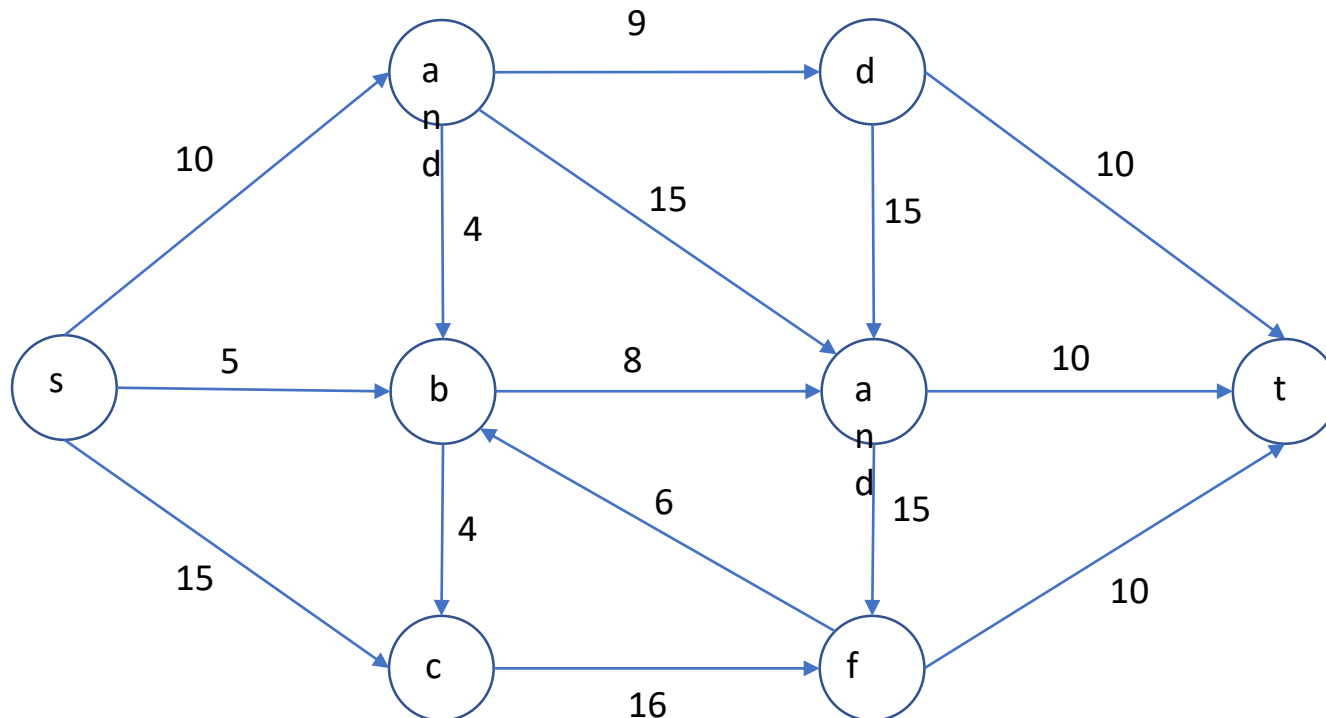
7) Flow distribution: GRAF

8) Minimum network cross-section?



Task 5

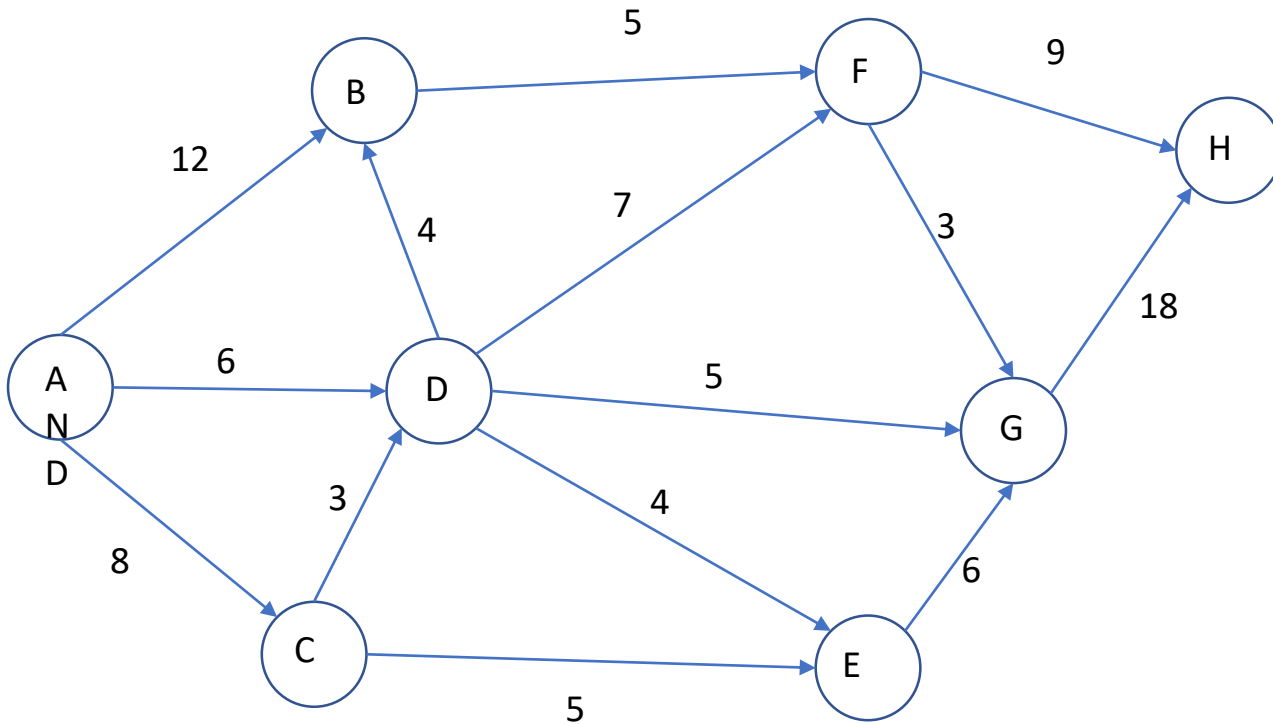
Determine the maximum number of trucks that we can send from point s to point t, taking into account road capacity. Mark the minimum cross-section of the network.





Task 6

Given a flow network defined by the graph:

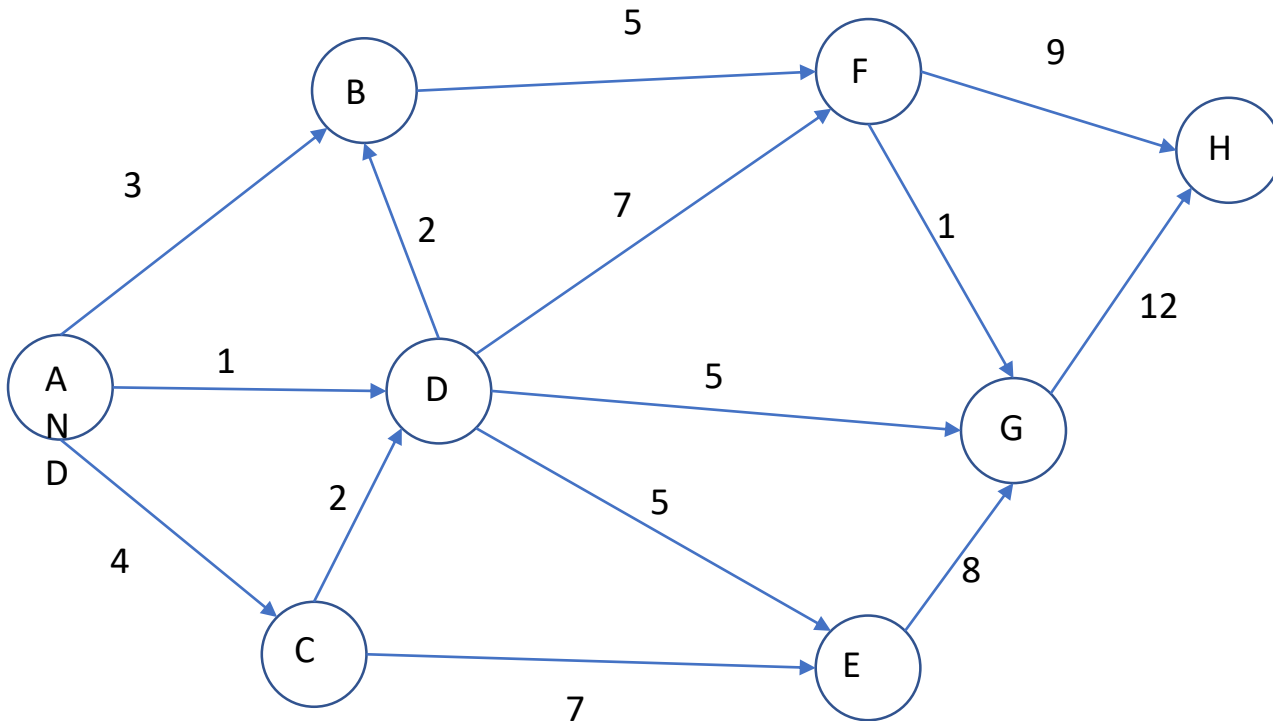


Determine the flow distribution in the network. Determine the maximum flow and determine the minimum cross-section of the network. Are there any connections in the network that can be eliminated without changing the flow? Define these connections



Task 7

Given a flow network defined by the graph:



Determine the flow distribution in the network. Determine the maximum flow and determine the minimum cross-section of the network. Are there any connections in the network that can be eliminated without changing the flow? Define these connections

Sources:

- Materials from the subject posted on the eNauczanie website : Z. Kędra
- Z. Jędrzejczyk, J. Skrzypek, K. Kukuła, A. Walkosz : Operational research in examples and tasks. PWN. Warsaw, 1996
- M. Glinka: Elements of operational research in transport. Radom University of Technology Publishing House. Radom, 2009
- Other books and textbooks on Operational Research available in the PG Library